



Localization for Anisotropic Sensor Networks

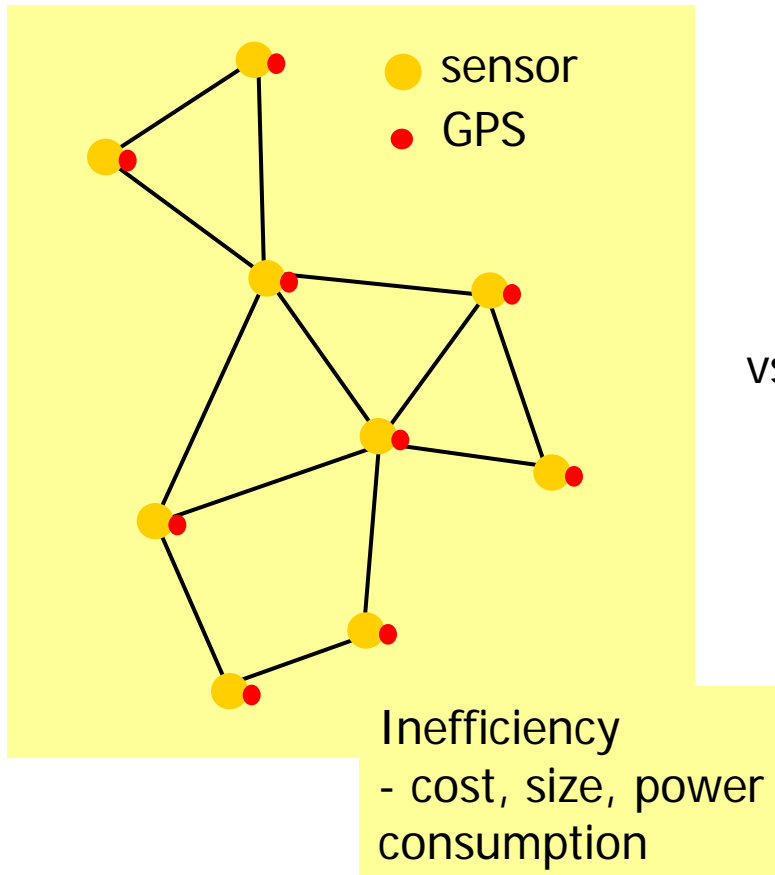
IEEE INFOCOM, March 15, 2005

Hyuk Lim and **Jennifer C. Hou**

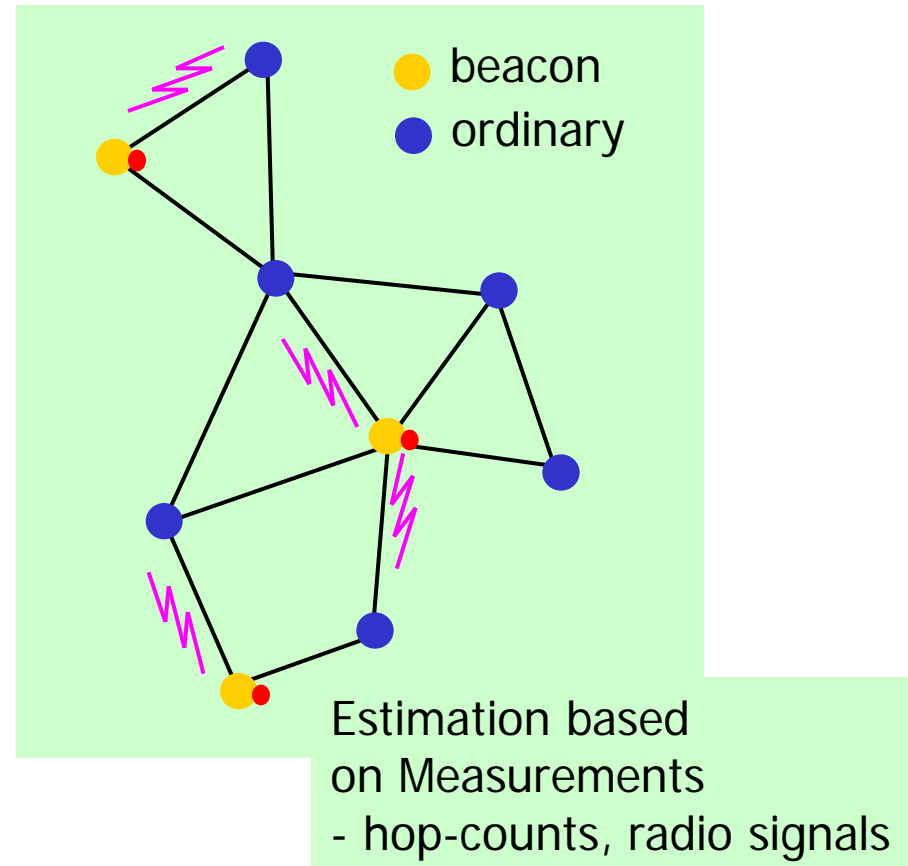
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Localization

- Determining **geographical locations** of sensor node in sensor networks.

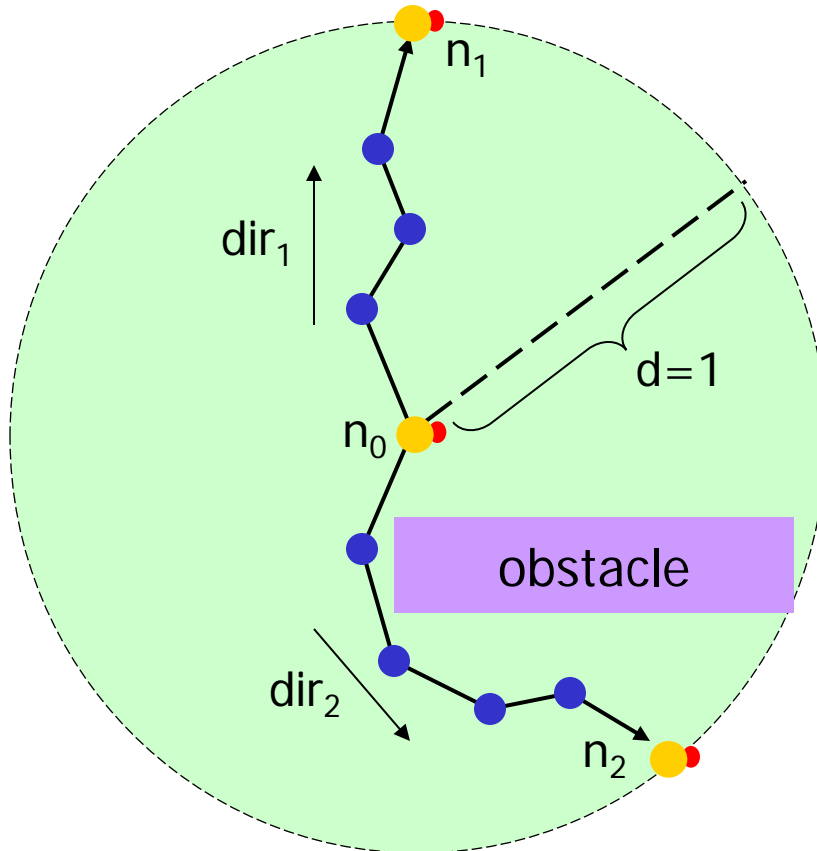


VS



Anisotropy

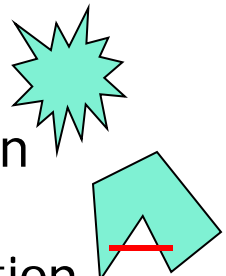
- Definition: “Having properties that **differ** according to the **direction** of measurement.”



Ratio of distance / hops
is anisotropic.

	distance	hops	ratio
dir ₁	1	4	1/4
dir ₂	1	5	1/5

- Irregular radio pattern
- Non-convex region
- Non-uniform distribution of beacon nodes





This paper ...

- Goal: To achieve **accurate** and **robust** estimation of geographical node locations in **anisotropic** sensor networks.
- Procedures:
 - Discovering anisotropic properties from measurements between beacon nodes.
 - Constructing a **proximity-distance map (PDM)** retaining as much topological information as possible in all directions of the measurements.
 - Estimating distances from a node without a GPS to beacon nodes and computing the location by triangulation.



Related Work

- Centroid localization - Bulusu et al., IEEE Pers. Commun. 2000
- **Adhoc positioning system (APS)** - Niculescu et al., Globecom 2000
 - DV-hop, DV-distance, and Euclidean propagation method.
 - In DV-hop, each beacon node computes the distance per hop by averaging the measurements from all the directions of beacon nodes

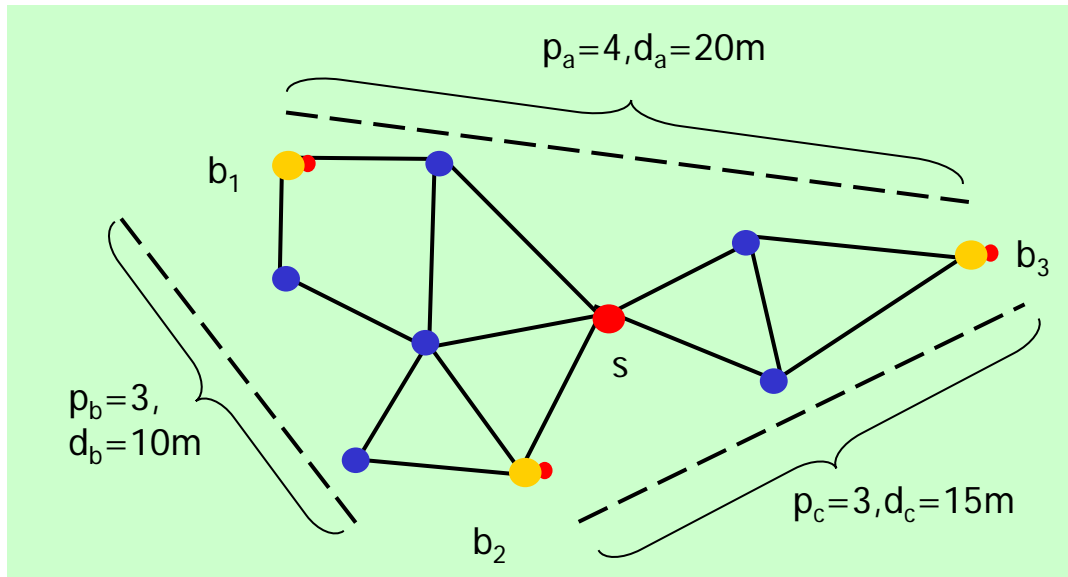
$$c_i = \frac{\sum_j \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{\sum_j h_j}, i \neq j \quad d_{ik} = c_i \times h_{ik}$$

- Amorphous localization - Nagpal et al., ISPN 2003
- APIT - He. et al., Mobicom 2003
- **Multidimensional Scaling (MDS) based algorithm**
 - Ji et al. INFOCOM 2004
 - Shang et al. Mobihoc 2003, INFOCOM 2004

Hundreds of papers in this topic ...

Proximity and Distance

- Proximity: quantitative measure that reflects the geographic distance.
- Example: proximity = number of hops



For b_1 ,

$$\text{proximity } \vec{p}_1 = [0, 3, 4]^T$$

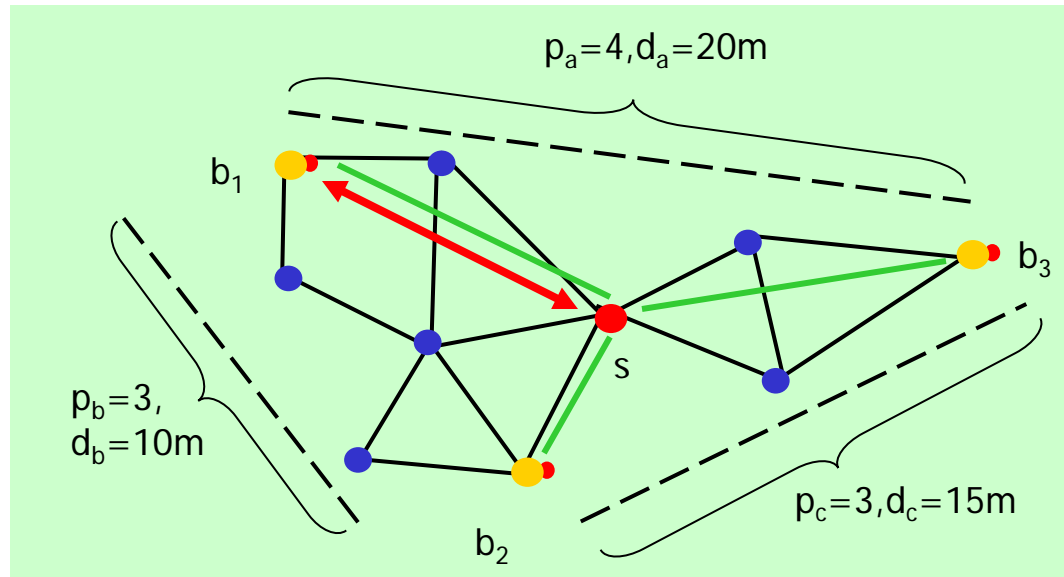
$$\text{distance } \vec{d}_1 = [0, 10, 20]^T$$

For s ,

$$\text{proximity } \vec{p}_s = [2, 1, 2]^T$$

$$\text{distance } \vec{d}_s = ?$$

Localization for Anisotropic Networks



DV-hop

Average ratio over all the directions

average distance
per one hop for b_1

$$c_1 = \frac{10 + 20}{3 + 4} = 4.28$$

distance between
 b_1 and s

$$d_{b_1s} = 4.28 \times p_{b_1s} = 8.56$$

Proposed method

Generalized proximity-distance function

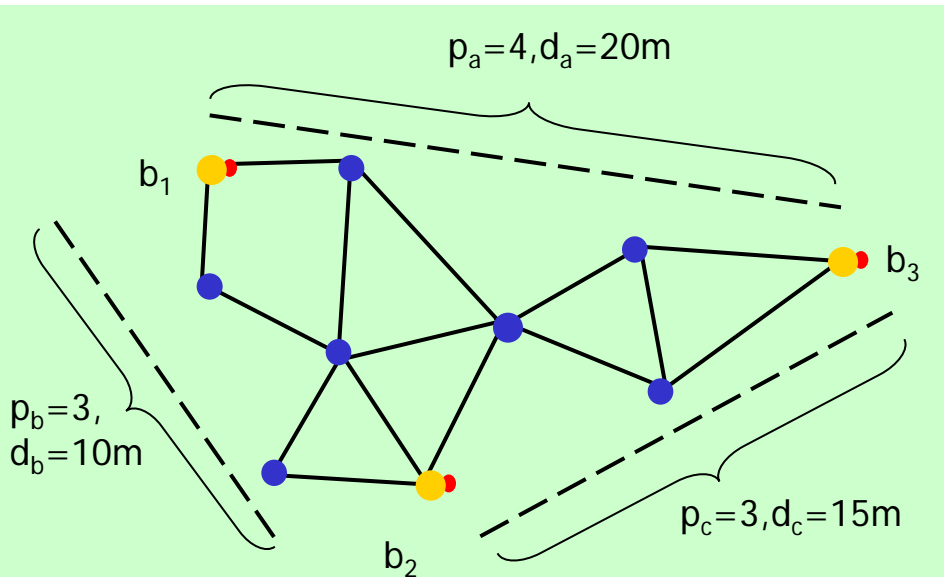
$$d_{b_1s} = t_{b_1b_1} p_{b_1s} + t_{b_1b_2} p_{b_2s} + t_{b_1b_3} p_{b_3s}$$

**Proximity
distance map**

$$T = \begin{bmatrix} t_{b_1b_1} & \cdots & t_{b_1b_M} \\ \vdots & \ddots & \vdots \end{bmatrix}$$

Proximity-Distance Map

- Construct proximity-distance map (PDM) from measurements between beacon nodes
- Least square solution: $T = DP^T(PP^T)^{-1}$
 - P and D are proximity and distance matrices.



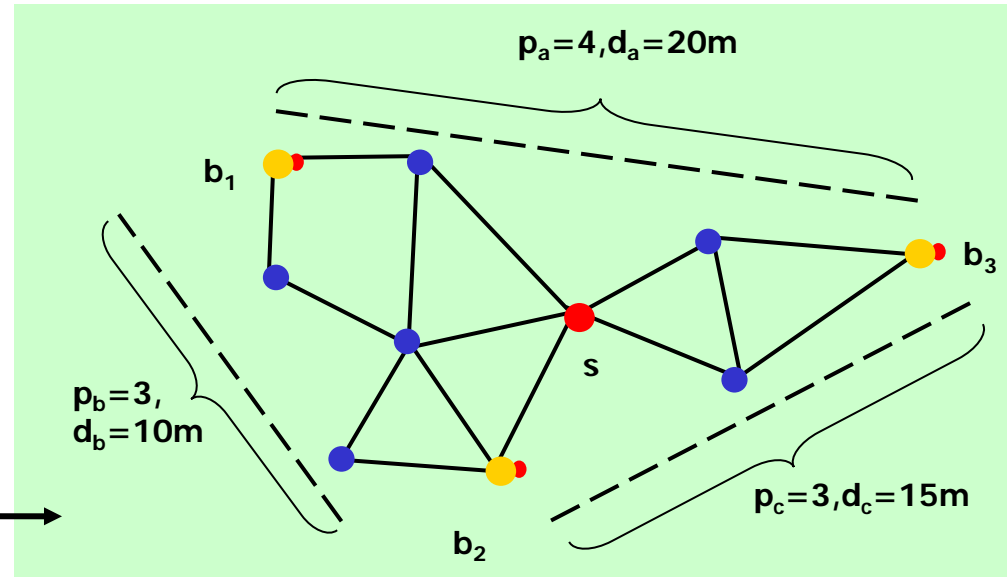
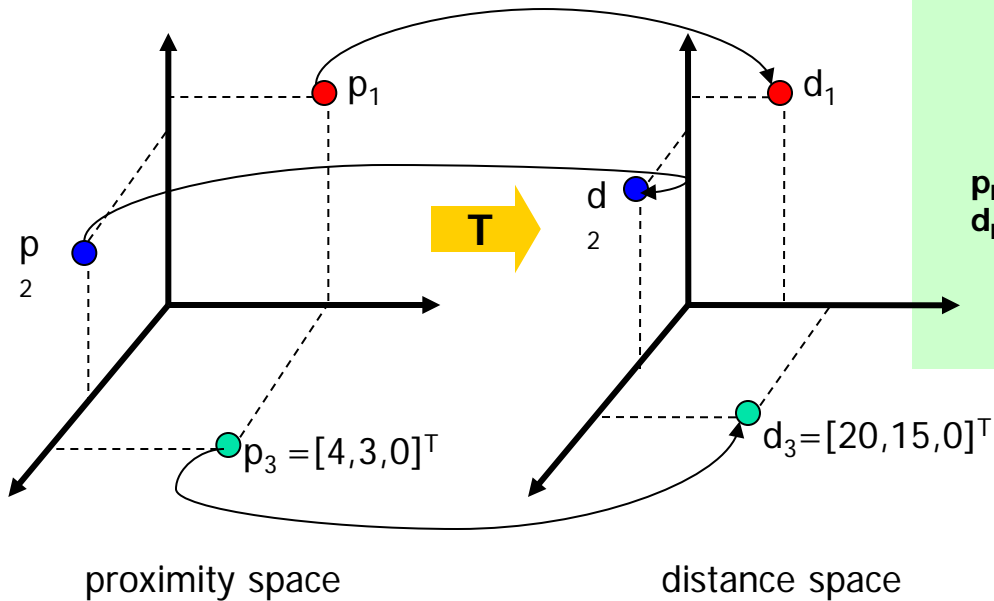
$$\begin{bmatrix} 0 \\ 10 \\ 20 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 15 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \\ 0 \end{bmatrix} = T \bullet \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 4.16 & 1.11 & -0.83 \\ 0.62 & 4.16 & -0.62 \\ 0.00 & 0.00 & 5.00 \end{bmatrix}$$

Distance to i^{th} beacon node
 $= T_{ij} \times \text{Proximity to } j^{\text{th}} \text{ beacon node}$

Distance Estimation for Sensors

PDM: Transformation matrix from a proximity embedding space to a distance embedding space.



for node s ,

$$\vec{d}_s = \begin{bmatrix} 7.77 \\ 4.16 \\ 10.0 \end{bmatrix} = T \bullet \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

Robust Computation of PDM

- Matrix inversion incurred by $T = DP^T(PP^T)^{-1}$
 - Singular value decomposition (SVD)

$$P = U \cdot \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \cdot V^T \quad \begin{array}{l} U = [u_1, \dots, u_M] \\ V = [v_1, \dots, v_M] \end{array} \quad \Sigma = \text{diag}(\sigma_1, \dots, \sigma_w)$$

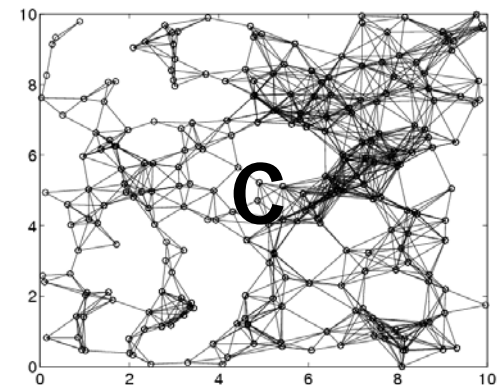
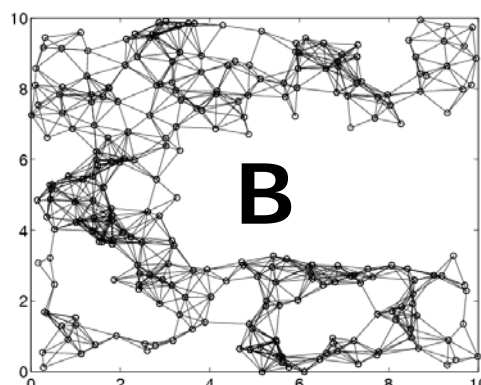
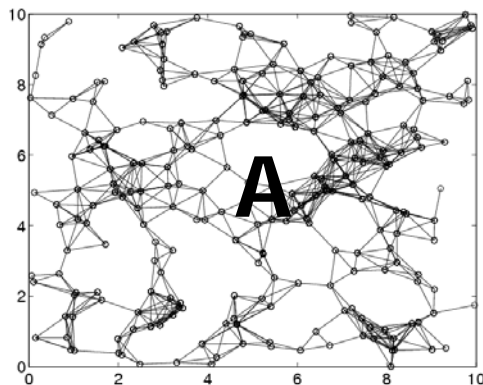
$$P^T(PP^T)^{-1} = V \cdot \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & 0 \end{bmatrix} \cdot U^T = \sum_{i=1}^w \frac{1}{\sigma_i} v_i u_i^T \quad \Sigma^{-1} = \text{diag}(\sigma_1^{-1}, \dots, \sigma_w^{-1})$$

- Truncated pseudo-inversion technique
 - Discarding components corresponding to small (near-zero) singular values by truncating terms at an earlier index $\gamma < w$.
 - $T = D \cdot \sum_{i=1}^{\gamma} \frac{1}{\sigma_i} v_i u_i^T$
 - Cumulative percentage method for selecting γ ($\tau^* = 0.98$).

$$\tau(\gamma) = \frac{\sum_{i=1}^{\gamma} \sigma_i}{\sum_{i=1}^w \sigma_i} \geq \tau^*$$

Simulation Results

- Network setup
 - Radio range = u and $1.3u$, Area = $10 \times 10u$
 - # of nodes = 250, # of beacon nodes = $4 \sim 30$.
- Algorithms: APS, MDS-based algorithm, PDM
 - MDS-based algorithm
 - Use SVD but take 2-3 components for obtaining 2-3 dimensional representation (locations).
 - Complexity $\sim O(N^3)$, N = total number of sensor nodes.
- Topologies



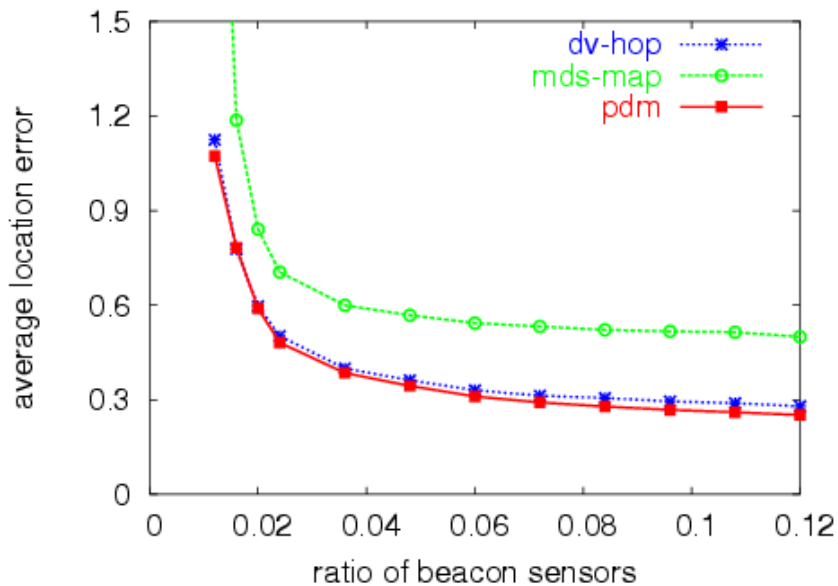
Performance in Isotropic Networks

Topology A

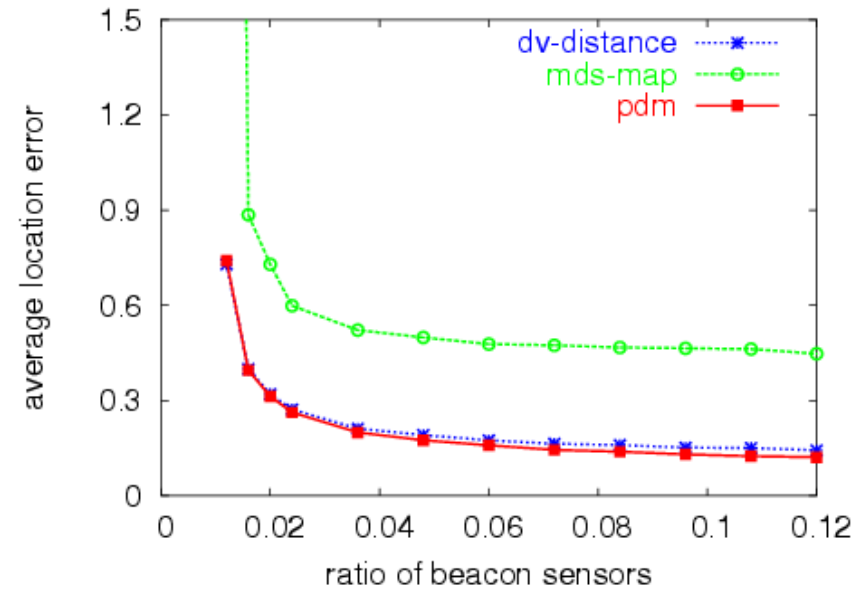
Uniform distribution of beacon and sensor nodes

High connectivity ($r=1.3u$)

Hop-count



Estimated distance

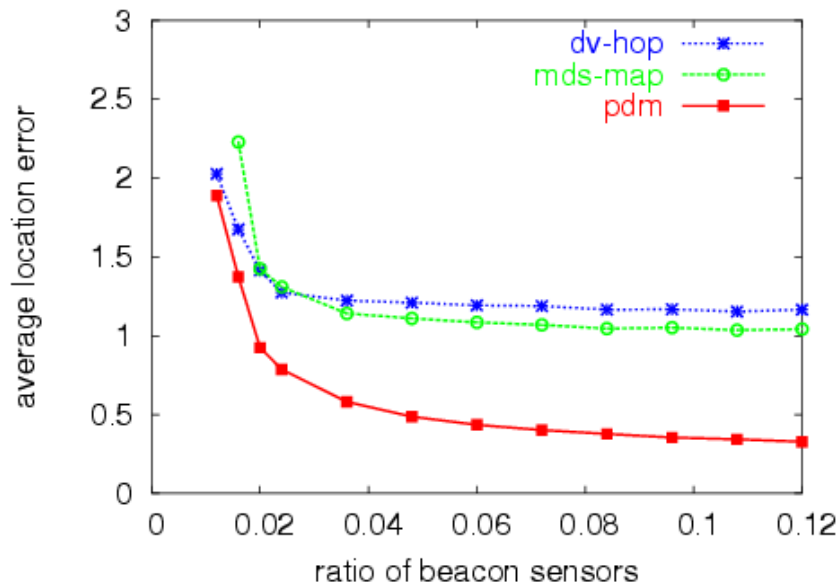


DV-hop (DV-distance) and PDM achieve the same performance in isotropic network using the same amount of measurement data.

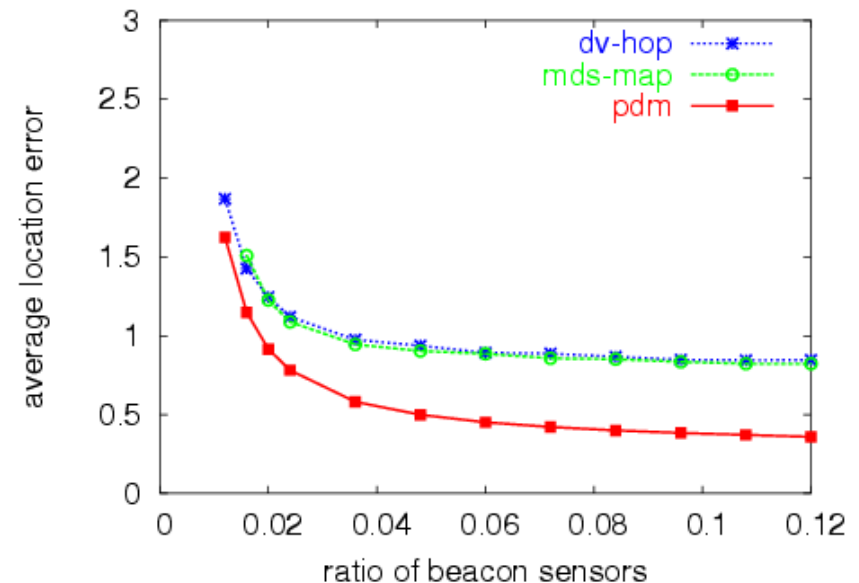
Performance in Anisotropic Networks

Topologies B and C
Uniform distribution of beacon
Proximity = Hop-count

Topology B



Topology C



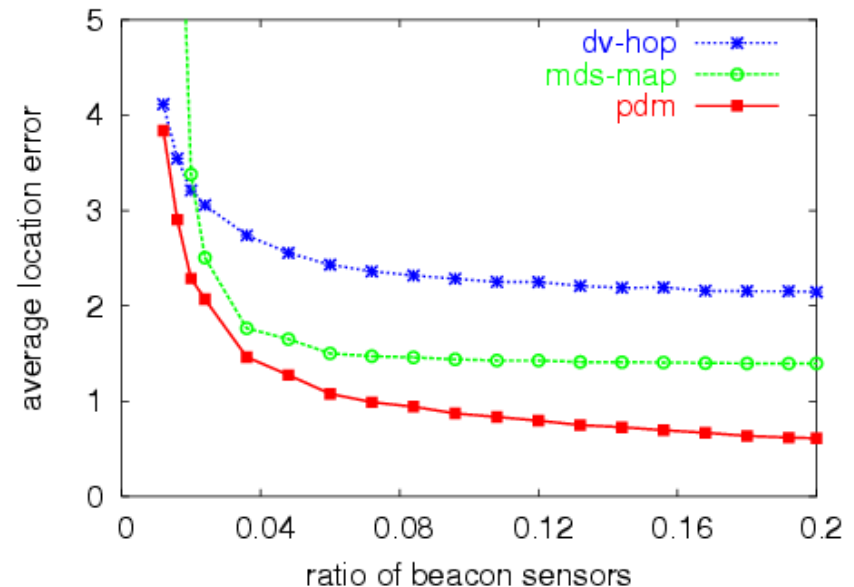
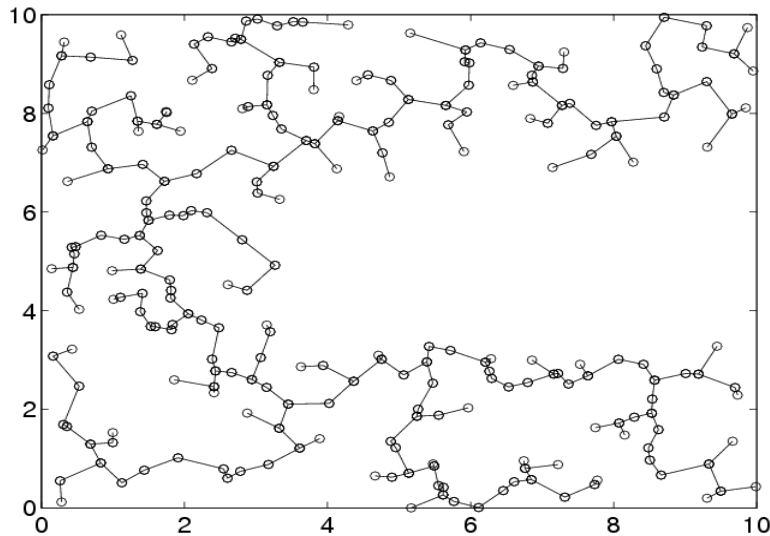
PDM achieves much better performance
in anisotropic sensor networks

Performance under Topology Control

Under Topology Control (Minimum Spanning Tree)

Low connectivity

Each node has a different radio range

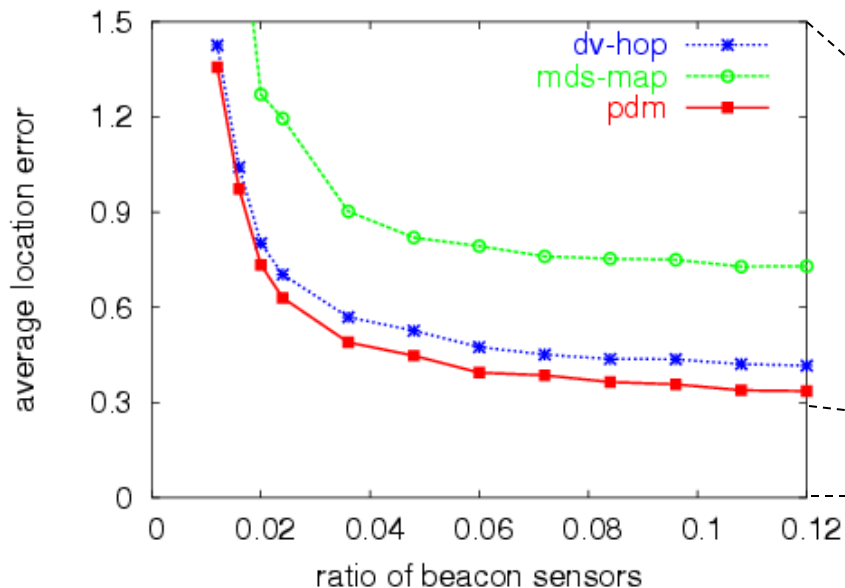


Under topology control, more beacon nodes are needed for accurate estimation. Still PDM achieves the best performance.

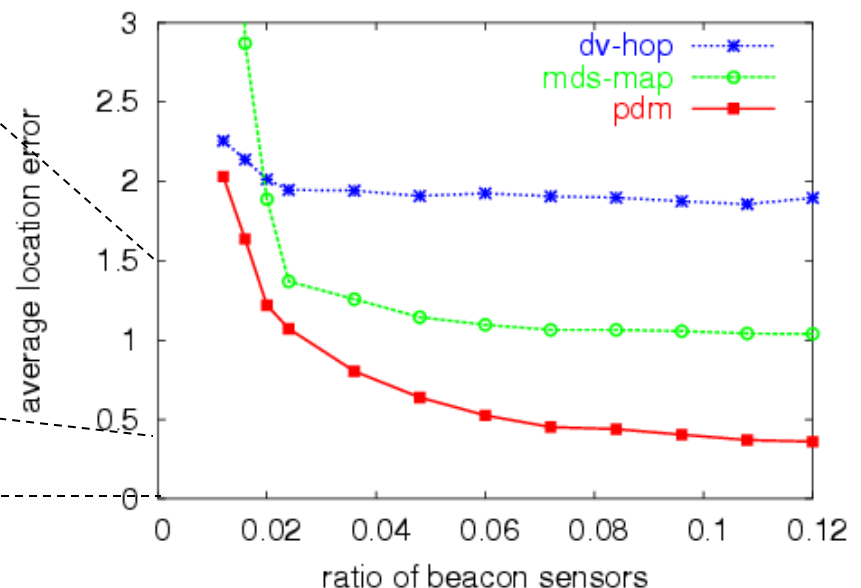
Non-uniform Distribution of Beacons

Topologies A and B
Non-Uniform distribution of beacon

Topology A



Topology B



PDM achieves almost the same performance even when beacon nodes are not uniformly distributed.



Summary

- PDM achieves accurate and robust estimation of locations in both isotropic and anisotropic sensor networks.
- In various scenarios, the achieved location errors in anisotropic sensor network were below 0.4 radio range as long as a ratio of beacon nodes exceeds a certain threshold.