Distributed Localization for Anisotropic Sensor Networks

Hyuk Lim and Jennifer C. Hou

Abstract

In this paper, we consider the issue of localization in anisotropic sensor networks. Anisotropic networks are differentiated from isotropic networks in that they possess properties that vary according to the direction of measurement. Anisotropic characteristics result from various factors such as the geographic shape of the region (non-convex region), the different node densities, the irregular radio patterns, and the anisotropic terrain conditions. In order to characterize anisotropic features, we devise a linear mapping method that transforms proximity measurements between sensor nodes into a geographic distance embedding space by using the truncated singular value decomposition-based (TSVD-based) pseudo-inverse technique. This transformation retains as much topological information as possible and reduces the effect of measurement noises on the estimates of geographic distances. We show via simulation that the proposed localization method outperforms DV-hop, DV-distance, and MDS-map, and makes robust and accurate estimates of sensor locations in both isotropic and anisotropic sensor networks.

Index Terms

Localization, Sensor networks, and singular value decomposition.

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I. INTRODUCTION

Driven by advances in MEMS micro-sensors, wireless networking, and embedded processing, ad-hoc networks of devices and sensors with (limited) sensing and wireless communication capabilities are becoming increasingly available for commercial and military applications such as environmental monitoring (e.g., traffic, habitat, security), industrial sensing and diagnostics (e.g., factory, appliances), critical infrastructure protection (e.g., power grids, water distribution, waste disposal), and situational awareness for battlefield applications. For these purposes, each sensor node collaborates with others in sensing, monitoring, and tracking events of interests by exchanging acquired data, usually stamped with the time and position information. If the data sent by a sensor node carries incorrect position information, it could be useless or even harmful. As such, localization — how each sensor node obtains its accurate position, even in the presence of different geographic shapes of the monitoring region (non-convex region), different node densities, irregular radio patterns, and anisotropic terrain conditions — has become an important and critical issue in deploying wireless sensor networks.

Localization for wireless sensor networks has been intensively studied in recent years. A simple approach of having all the sensor nodes equipped with a global positioning system (GPS) does not suffice because of the size, cost, and power consumption constraints of sensor nodes. Instead, most localization methods determine the positions of *unknown* sensor nodes under the assumption that a small portion of sensor nodes, called *beacon nodes,* are aware of their positions by means of manual configuration or GPS [1]-[12]. In these methods, each sensor node estimates (based either ranging techniques or proximity measurements) its distances to beacon nodes, and calculates its position by triangulation/lateration techniques. Refinement can be made to iteratively improve the accuracy of these localization methods, by, for example, gradually adjusting the node position so as to minimize the discrepancy between the calculated Euclidean distances and the measured distances to its neighboring nodes [5], [6].

One underlying assumption used in most localization methods is that the network topology is isotropic, i.e., the properties of proximity measurements are identical in *all* directions. Unfortunately, this assumption often does not hold in practice, due to the geographic shape of the region (non-convex region), the different node densities, the irregular radio patterns, and the anisotropic terrain conditions. As a result, their performance degrades severely in anisotropic sensor networks. For example, in one of the pioneering methods, APS [3], each beacon node computes the average distance per hop by dividing the sum of distances to the other beacon nodes by the sum of hop-counts, without taking into account of the fact that the per-hop distance may be different in different directions, due to terrains, obstacles, and/or other effects. A sensor node that does not know its location estimates its distance to a beacon node, by multiplying the average per-hop-distance of the beacon node by the hop-count to the beacon node (measured by the sensor node).

Recently, several methods have been proposed for anisotropic networks, among which the multidimensional scaling (MDS) based methods [11], [12] may have received the most attention. By assuming that the network is locally isotropic in small regions, they establish local maps based on the MDS technique in small regions, and merge local maps into a global map covering the entire sensor network area. Although these "divide and conquer" methods further improves the accuracy of localization under certain cases, their performances are quite susceptible to the choice of the size of small regions. As a matter of fact, this parameter depends greatly on the terrain conditions and other factors that affect the isotropy of the network.

In this paper, we present a new technique to analyze the relationship between the geographic distance and the proximity between sensor nodes in anisotropic networks. Conceptually, localization can be considered as an embedding problem that maps the set of objects into an embedding space. In Lipschitz embeddings, a coordinate space is defined such that each axis corresponds to a *reference set* of objects, and the coordinate values of an object o are the distances from o to the reference objects [13], [14], [15]. Based on this concept, each sensor node has two coordinates in Lipschitz embedding spaces that correspond, respectively, to the proximity measure and the Euclidean distance between itself and beacon nodes.

We derive an optimal linear transformation that projects one embedding space (that is built upon proximity measures) into the geographic distance space by using the singular value decomposition (SVD) technique. The $(i, j)^{th}$ element of the transformation matrix represents the effect of proximity to the j^{th} beacon node on the geographic distance to the i^{th} beacon node. The distance to a beacon node is computed by a weighted sum of proximities to all the beacon nodes in *all* directions. Moreover, by introducing a truncation method to SVD, the proposed method reduces the effect of noise in the transformation process, while keeping as much topological information as possible. Finally, we show via simulation that as compared to

MDS-based localization methods, the proposed localization method makes robust and accurate estimates of node locations in both isotropic and anisotropic sensor networks.

The rest of the paper is organized as follows: In Section II, we provide preliminary material and formulate the localization problem. In Section III, we give a summary of related work in the literature. In Section IV-V, we first introduce optimal linear transformation from the proximity space into the geographic distance space, and then elaborate on system implementation issues. Following that, we present in Section VI experimental results, and conclude the paper in Section VII.

II. BACKGROUND

Localization Problem*:* The localization problem we consider is as follows: Given the *proximity measures* to beacon nodes, determine the unknown locations of sensor nodes, where the proximity between two nodes is defined as a quantitative measure that reflects the geographic distance. For example, in range-free sensor networks, network characteristics such as the number of hops are adequate candidates as the proximity measure.

Consider a sensor network $\mathcal{S}$ with M beacon nodes and N (non-beacon) nodes with unknown positions. (For notational convenience, we term the nodes with unknown positions as *unknown* nodes.) The locations of beacon nodes and unknown nodes are denoted as $\mathbf{x}_i \in \mathbb{R}^d$ in d-dimensional space for $i = \{1, \cdots, M\}$ and $i = \{M+1, \cdots, M+N\}$, respectively. The geographic distance between two nodes, $\mathbf{x}_i$ and $\mathbf{x}_j$ is then defined by the Euclidean distance:

$$d_{ij} = f_d(\mathbf{x}_i, \mathbf{x}_j) := \sqrt{\sum_{k=1}^{d} (x_{ik} - x_{jk})^2}, \quad (1)$$

where x_{ik} and x_{jk} are the k^{th} coordinates of $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively. Let p_{ij} be the proximity measure between the i^{th} node and the j^{th} node. Then the localization problem can be formally stated as

Given: $\mathbf{x}_i,\ p_{ij},$ and p_{si} for $i, j \in \{1, \cdots, M\}$,

Estimate: $\mathbf{x}_s$ for a sensor node s.

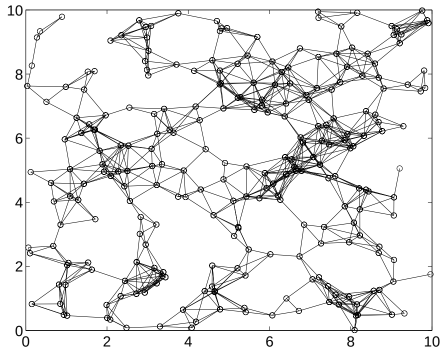
Namely, under the assumption that the locations $\mathbf{x}_i$ of the beacon nodes are known, the problem is to estimate, with the use of the proximities p_{ij} and p_{si} for $i, j \in \{1, \cdots, M\}$, the geographic position $\mathbf{x}_s$ of the sensor node s.

Definition of Isotropy: For a sensor network $\mathcal{S}$, we assume that there exists a certain mapping function, $f_p : \mathbb{R}^{2d} \longrightarrow \mathbb{R}$, that describes the mapping from the geographic locations ($\mathbf{x}_i$ and $\mathbf{x}_j$) to the measured proximity p_{ij} for each pair of sensor nodes, where the proximity is written as $p_{ij} = f_p(\mathbf{x}_i, \mathbf{x}_j)$. If the mapping $f_p(\mathbf{x}_i, \mathbf{x}_j)$ is a function of the Euclidean distance between $\mathbf{x}_i$ and $\mathbf{x}_j$, the sensor network is said to be *isotropic*, i.e. $p_{ij} = f_p(\mathbf{x}_i, \mathbf{x}_j) = g_p(d_{ij})$, $\forall\ i, j \in \{1, \ldots, M + N\}$ and $g_p : \mathbb{R} \longrightarrow \mathbb{R}$.

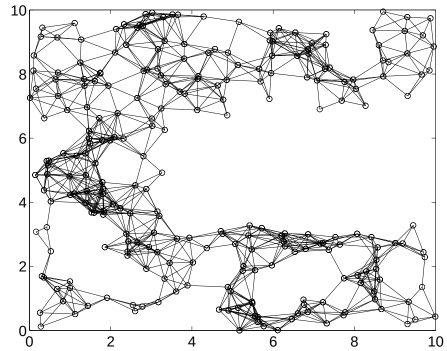
In practice, the proximities measured by a sensor node to the others often differ in different directions. This implies that the proximity between a pair of sensor nodes depends greatly on the distinct locations of these sensor nodes, and the sensor network is *anisotropic*. For instance, if the proximity is defined by the minimum hop-count obtained by flooding probing packets, and if sensor nodes are scattered in a non-convex region, the path between a pair of sensor nodes may not be a straight line and detours around the region. This results in a larger proximity between the sensor nodes than that in a convex region. Similarly, in a weakly connected sensor network, the geographic distance may be shorter than the product of the hop-count and the transmission range, as intermediate nodes may not exist on the straight line between the two nodes. That is, a loosely populated sensor network is likely anisotropic.

Figure 1 gives several examples of isotropic/anisotropic sensor networks. (We will use these networks both for the subsequent discussion and for the simulation study.) Figure 1 (a) gives an isotropic sensor network, where 250 sensor nodes (each with a radio range of r) are uniformly distributed within a square area. For notational convenience, we normalize the distance with the radio range r, i.e., the distance is measured in units of $u = r$. The square area is of size $10u \times 10u$. Figure 1 (b) and (c) give two possible anisotropic sensor networks. In Fig. 1 (b), sensor nodes enclose a circular obstacle in the right half plane, and an anisotropic network results because of geographic structures. In this case, even though the geographic distances of two pairs of nodes are the same, their proximities can be quite different. In Fig. 1 (c), sensor nodes in the left half plane have a radio range of $r_1 = u$, whereas those in the right half plane have a radio range of $r_2 = 1.3u$. An anisotropic network results because of different radio ranges (due to, for example, terrain and foliage effects). This difference also makes the ratio of the geographic distance to the hop-count different in different regions.

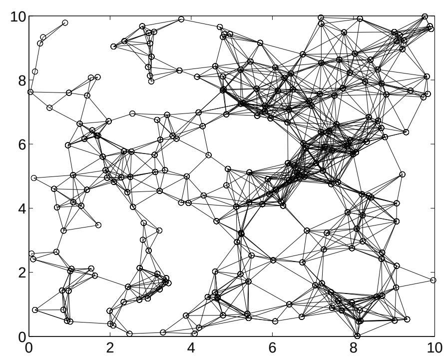
In anisotropic sensor networks, in order to obtain accurate localization results, it is necessary to compensate for the anisotropic properties by gathering and utilizing information on the



(a) Topology $\mathcal{A}$: Isotropic network



(b) Topology $\mathcal{B}$: Anisotropic network due to geographic structures



(c) Topology $\mathcal{C}$: Anisotropic network due to different radio ranges

Fig. 1. Sensor network topologies used in the simulation study.

relationship between the geographic distances and the measured proximities in as many directions as possible.

III. Related Work

A. Generic Approaches

Bulusu *et al.* [1] attempted to reduce the use of GPS by placing multiple nodes (beacon nodes) with overlapping coverage regions at known locations. The authors proposed a simple localization method that determines the location of a sensor node as the centroid of the locations of its neighboring beacon nodes. Doherty *et al.* [2] formulated the localization problem as a convex optimization problem with proximity constraints imposed by known connections. The problem was then solved in a centralized manner.

Niculescu *et al.* [3] proposed a distributed positioning algorithm, called *ad-hoc positioning system* (APS), in which three different propagation methods were investigated, i.e., *DV-hop*, *DV-distance*, and *Euclidean* schemes. The DV-hop scheme employs distance vector exchange. Each node exchanges distance tables that contain the locations of, and the hop-counts to, beacon nodes with its neighboring nodes. Once a beacon node obtains these distance tables from other beacon nodes, it estimates an average distance per hop, and exploits it to estimate the geographic distance from each unknown node to the beacon node. The unknown node estimates its location by performing lateration, i.e., a simplified version of the GPS triangulation. The DV-distance scheme employs the geographic distance measured with the use of radio signals, rather than hop-counts. Finally the Euclidean scheme relies on the geometry of neighboring sensor nodes to estimate the geographic location.

Savvides *et al.* proposed an iterative multilateration method in [4] and its improved version with the Kalman filter based position refinement process in [6]. Each node simply calculates an initial estimate of its location based on geometric constraints, and updates the estimate by the iterative multilateration. Savarese *et al.* [5] proposed an algorithm split into two pahses: the *start-up* phase and the *refinement* phase. An initial position of a node is obtained in the start-up phase and is gradually adjusted in the refinement phase by using the measured ranges between its neighboring nodes. Nagpal *et al.* [7] proposed a coordinate formation algorithm that consists of a gradient descent method for estimating the distances to neighboring nodes and a multilateration method for estimating the locations. This algorithm achieves estimate accuracy within 20% of the radio range in a reasonable simulation environment. He *et. al* [8] proposed a simple, area-based localization technique that does not require expensive lateration algorithms. Each node chooses three beacon nodes from all neighboring beacon nodes, forms the triangles by connecting these three beacon nodes, and calculates the center of the intersection of all the triangles to determine its position.

Most of the proposed positioning algorithms including [1]-[8] work well in isotropic sensor networks. However, their performance severely degrades in anisotropic networks as a result of not taking into account of the anisotropic properties.

B. Multidimensional Scaling (MDS) Based Approaches

Several novel methods using multidimensional scaling (MDS) were recently proposed for localization in sensor networks [9], [10], [11], [12]. Multidimensional scaling is a data analysis technique used to visualize proximity of a set of objects in a low dimensional space. Let $\mathbf{P}$ be a proximity matrix, whose ij^{th} element is the proximity measured between the i^{th} and j^{th} sensor nodes. The squared matrix $\mathbf{P}\mathbf{P}^T$ is shifted to the center of the matrix $\mathbf{P}$, and is decomposed by similarity transformation. Then, by selecting the eigenvectors associated the first m largest eigenvalues, these localization methods obtain an m dimensional space representation (m is usually 2-3), called a *relative map*. Locations in the relative map are relative to each other, and hence have to be rotated, shifted, and reflected in order to coincide with the geographic locations of sensor nodes.

Raykar *et al.* [9] formulated a localization problem for sound sensors and actuators as a non-linear least square minimization problem. The authors suggested to use the coordinates obtained by MDS as the initial guess to mitigate the local minima problem.

Shang *et al.* [10] proposed a MDS-based localization method, called *MDS-map*, that works well with connectivity information. However, *MDS-map* requires availability of global connectivity information for all the sensor nodes (in order to calculate the similarity transformation), and is a centralized method with complexity O(n^3), where n is the total number of sensor nodes. Moreover, *MDS-map* does not seem to outperform the previous methods, when the number of beacon nodes are large.

To eliminate the need for global connectivity information and centralized computation, Shang *et al.* [11] proposed an improved version of *MDS-map*, called *MDS-map (P)*. Each node performs MDS with the connectivity information to its neighbor nodes and obtains a local (relative) map. These local maps are then merged together to form a global (relative) map. The global map has to be aligned, by a linear transformation, in order to construct a geographic (absolute) map.

Ji and Zha [12] proposed to use the *scaling by majorizing a complicated function* (SMACOF) to obtain weighted MDS iteratively, when a portion of the pairwise proximity information is not available. The relative maps are calculated in small groups of sensor nodes and are merged in a distributed fashion. The authors proposed an incremental and distributed method to align the relative map to the geographic map.

Although the MDS-map method [10] leverages global connectivity information, it does not give better performance in anisotropic sensor networks than the latter two methods [11], [12] that establish small local maps and merge them to construct a global map. This is because MDS used in aforementioned approaches leave out significant information by using two (or three) eigenvectors of the n eigenvectors obtained from the similarity transformation. This implies that two (or three) dimensions are not sufficient to catch the anisotropic properties. Namely, the two (or three) eigenvectors selected in the MDS process retain only two (or three) principal components of the proximity information, and other significant information including anisotropic network properties are essentially left out. The latter two methods [11], [12], on the other hand, divide an anisotropic sensor network into a number of small regions, each of which is considered to be locally isotropic. Relative, local maps are then established, and merged into a global map. As a result, the anisotropic characteristics can be better retained in the global map. The downside of these two approaches is, however, the performance is quite susceptible to the choice of an appropriate region size and the origin of the global map. The performance is also affected by error propagation during the merging process.

C. Our Proposed Method

Our proposed method bears the similarity with the MDS-based methods in that it uses the singular value decomposition (SVD) technique to analyze the proximity matrix. However, it differs in several fundamental aspects:

- **Accurate characterization**: We employ SVD to analyze the relationship between the geographic distances and the proximities, with the objective of retaining as much anisotropic characteristics as possible.
- **Less computational complexity**: In the proposed method, SVD is applied to the proximity matrix only between beacon nodes, but not all the nodes. Although SVD has computational complexity of O(n^3), the parameter n refers to the number of beacon nodes. An unknown sensor node simply computes its geographic distance to beacon nodes by matrix multiplication.
- **Simple protocol operations**: The corresponding protocol in the proposed method is similar to that in APS [3]. Unlike the MDS-based approaches, it requires neither global topology

information, partition of the area into small regions for generating relative maps, nor global coordination and integration for a global map.

Our objective is to estimate, based on proximity measurements, the geographic distances from unknown sensor nodes to beacon nodes in anisotropic sensor networks. The geographic locations of sensor nodes are determined by lateration algorithms [3], [4], [7], and their estimation accuracy can be further improved by refinement techniques [5], [6].

IV. Proximity Characterization

In this section, we present our theoretical base for proximity characterization in wireless sensor networks. This relates to inferring network topology based on the geometric structure or other network attributes such as the hop-count. For this purpose, we analyze the proximities measured between beacon nodes with known geographic locations, and derive optimal linear transformation, called *proximity-distance map* (PDM), that describes the relationship between the proximities and the geographic distances in anisotropic sensor networks.

A. Embedding Spaces in Localization

The proximities measured from a (beacon or non-beacon) node to beacon nodes define its coordinate in a linear system. Given that there exist M beacon nodes, the coordinate of a node s_i in an M-dimensional Lipschitz embedding space [14], [15], is represented by the proximity vector:

$$\mathbf{p}_i = [p_{i1}, \cdots, p_{iM}]^T$$

where p_{ij} is the proximity measured by the i^{th} node to the j^{th} node and $p_{ii} = 0$. The overall embedding space can be represented by an M-by-M proximity matrix $\mathbf{P}$, whose i^{th} column is the coordinate of node s_i:

$$\mathbf{P} = [\mathbf{p}_1, \cdots, \mathbf{p}_M].$$

Here $\mathbf{P}$ is a square matrix with zero diagonal entries.

Similarly, we define the geographic distance vector and matrix as

$$\mathbf{l}_i = [l_{i1}, \cdots, l_{iM}]^T,$$

and

$$\mathbf{L} = [\mathbf{l}_1, \cdots, \mathbf{l}_M].$$

The geographic matrix $\mathbf{L}$ is an M-by-M symmetric square matrix with zero diagonal entries.

B. Proximity-Distance Map (PDM)

Now we derive optimal linear transformation $\mathbf{T}$, called *proximity-distance map* (PDM), that gives a mapping from the proximity matrix $\mathbf{P}$ to the geographic distance matrix $\mathbf{L}$. Note that $\mathbf{T}$ is an M-by-M square matrix. Each row vector $\mathbf{t}_i$ of $\mathbf{T}$ can be obtained by minimizing the following square error:

$$\begin{aligned} e_i &= \sum_{k=1}^{M} (l_{ik} - \mathbf{t}_i \mathbf{p}_k)^2 \\ &= ||\mathbf{l}_i^T - \mathbf{t}_i \mathbf{P}||^2. \end{aligned}$$

The least-square solution for the row vector $\mathbf{t}_i$ is

$$\mathbf{t}_i = \mathbf{l}_i^T \mathbf{P}^T (\mathbf{P}\mathbf{P}^T)^{-1}.$$

As a result, PDM is defined as

$$\mathbf{T} = \mathbf{L}\mathbf{P}^T (\mathbf{P}\mathbf{P}^T)^{-1}. \tag{2}$$

Remark 1 *The element t_{ij} of $\mathbf{T}$ represents the effect of the proximity to the j^{th} beacon node on the geographic distance to the i^{th} beacon node. Note that the main diagonal t_{ii} of $\mathbf{T}$ can be considered as scaling factors roughly approximating the mapping from the proximity to the geographic distance. The geographic distance from a node to a beacon node is specified as a weighted sum of proximities to all the beacon nodes.*

Note that as PDM retains all the proximity characteristics to all beacon nodes in all directions, it can precisely characterize the anisotropic relationship between proximities and geographic distances.

C. Calculation of PDM

We derive a numerically stable form of Eq. (2) with the use of the *singular-value decomposition* (SVD) [16]. Let the singular-value decomposition of $\mathbf{P}$ be expressed as

$$\mathbf{P} = \mathbf{U} \cdot \begin{bmatrix} \mathbf{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} \cdot \mathbf{V}^T, \tag{3}$$

$\mathbf{U}$ and $\mathbf{V}$ are column and row orthogonal matrices:

$$\mathbf{U} = [\mathbf{u}_1, \cdots, \mathbf{u}_M],$$
$$\mathbf{V} = [\mathbf{v}_1, \cdots, \mathbf{v}_M],$$

and $\mathbf{\Sigma}$ is a diagonal matrix:

$$\mathbf{\Sigma} = diag(\sigma_1, \cdots, \sigma_W),$$

where the subscript *w* is the rank of matrix $\mathbf{P}$, and σ_i's are singular values of $\mathbf{P}$ in the decreasing order (i.e., $\sigma_1 \geq \cdots \geq \sigma_W > 0$). Then the matrix $\mathbf{P}^+$, called *pseudo-inverse* or the *Moore-Penrose generalized inverse* of $\mathbf{P}$, is defined as

$$\begin{aligned} \mathbf{P}^+ &= \mathbf{P}^T(\mathbf{P}\mathbf{P}^T)^{-1} = \mathbf{V} \cdot \begin{bmatrix} \mathbf{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \cdot \mathbf{U}^T \\ &= \sum_{i=1}^{W} \frac{1}{\sigma_i} \mathbf{v}_i \mathbf{u}_i^T. \end{aligned} \tag{4}$$

With the use of $\mathbf{P}^+$, PDM can be simply expressed as $\mathbf{T} = \mathbf{L}\mathbf{P}^+$.

Caution should be taken in calculating and using the pseudo-inverse of $\mathbf{P}$. If the proximity measurements cannot be accurately made, the noise introduced in the measurement may be excited because of the terms that contain reciprocals of small, near-zero singular values in Eq. (4). To reduce such effects, we use the *truncated pseudo-inverse* method described in [17], [18], in which small singular values are simply discarded by truncating the terms at an earlier index $\gamma < w$. That is, instead of using $\mathbf{\Sigma}$ in Eq. (4), we use

$$\mathbf{\Sigma}_\gamma = diag(\sigma_1, \cdots, \sigma_\gamma),$$

and the truncated pseudo-inverse of $\mathbf{P}$ can be written by

$$\mathbf{P}_\gamma^+ = \sum_{i=1}^{\gamma} \frac{1}{\sigma_i} \mathbf{v}_i \mathbf{u}_i^T. \tag{5}$$

While truncating higher terms in Eq. (4) reduces the adverse effect of measurement noises, it may also result in significant loss of anisotropic information. To determine an adequate index γ, we use the following criterion: the percentage accounted for by the first k singular values is defined by

$$\tau_k = \frac{\sum_{i=1}^{k} \sigma_i}{\sum_{i=1}^{W} \sigma_i}.$$

One may pre-determine a cut-off value, τ^* of cumulative percentage of singular values, and calculate γ to be the smallest integer such that $\tau_\gamma \geq \tau^*$. We usually set τ^* to 0.98.

Remark 2 *PDM reconstructs an embedding space for geographic distances using proximities measured between beacon nodes, i.e.,* $\tilde{\mathbf{L}} = \mathbf{T}\left[\mathbf{p}_1, \cdots, \mathbf{p}_M\right] = \mathbf{L}\mathbf{P}_\gamma^+\mathbf{P} \approx \mathbf{L}\mathbf{P}^+\mathbf{P} = \mathbf{L}$.

A sensor node with an unknown position can obtain its proximity vector $\mathbf{p}_s$ by counting, for example, the hop-counts to the beacon nodes. It then obtains the estimate of its geographic distances to the beacon nodes by multiplying $\mathbf{p}_s$ with PDM:

$$\tilde{\mathbf{l}}_s = \mathbf{T}\mathbf{p}_s = \mathbf{L}\mathbf{P}_\gamma^+\mathbf{p}_s. \tag{6}$$

We will discuss in Section V the protocol operations for beacon nodes to gather information for calculating $\mathbf{T}$, and for sensor nodes to obtain $\mathbf{T}$.

D. Performance Evaluation

We evaluate the performance of PDM with respect to the estimation accuracy of geographic distances between sensor nodes in anisotropic sensor networks. The three network configurations depicted in Figure 1 are used. We assume, for simplicity, that beacon nodes flood probing packets to the entire sensor network and the proximity is measured in the number of hops. We compare the performance of PDM with that of DV-hop [3], in which each beacon node b calculates the average geographic distance per hop-count as

$$c_b = \frac{\sum_i^M f_d(\mathbf{x}_b, \mathbf{x}_i)}{\sum_i^M p_{bi}},$$

and the geographic distance between a beacon node b and a sensor node s is calculated as $l_{sb} = c_b\ p_{sb}$. In each simulation run, we calculate the error index as

$$\mathcal{E}_d = \frac{\sum_{i=1}^{M+N}\sum_{j=1}^{M}\left|l_{ij} - \tilde{l}_{ij}\right|}{M(M+N)u},$$

where M and N are the numbers of beacon nodes and unknown nodes, respectively.

First, we evaluate the performances of DV-hop and PDM with respect to different radio ranges ($u \leq r \leq 2u$) in the case of M = 10. As shown in Fig. 2, the average errors of DV-hop and PDM do not differ significantly in isotropic networks (Topology $\mathcal{A}$). On the other hand, PDM gives significantly smaller average errors in anisotropic networks (Topology $\mathcal{B}$). An interesting

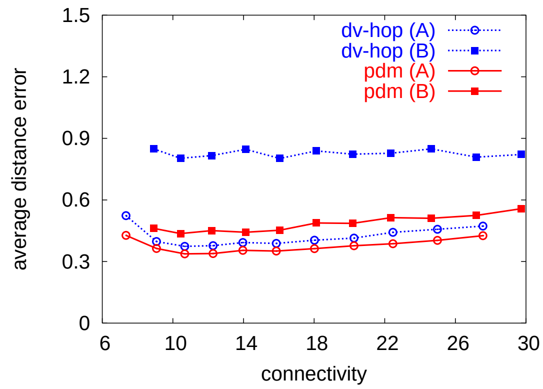
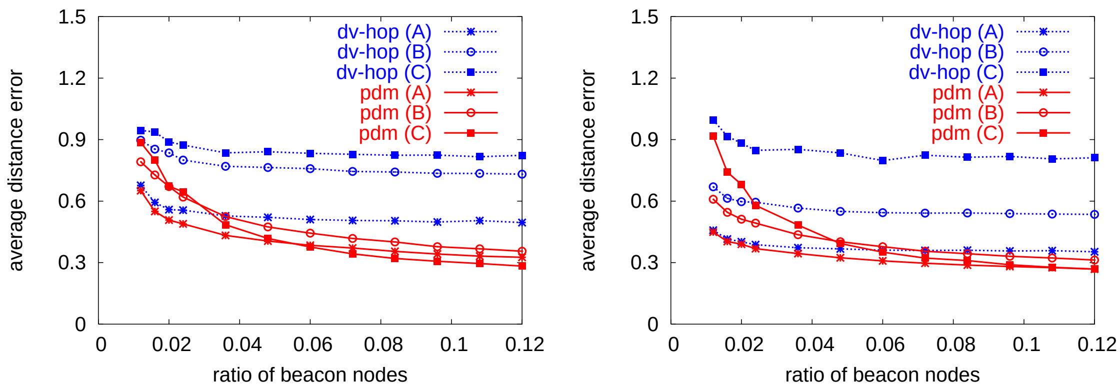


Fig. 2. Average geographic distance errors versus different radio ranges ($u \leq r \leq 2u$) when the number of beacon nodes is 10 in Topologies $\mathcal{A}$ and $\mathcal{B}$. The x-axis is the number of neighbors (as r changes from u to $2u$).



(a) $r = u$

(b) $r = 1.3u$

Fig. 3. Average geographic distance errors when the number of beacon nodes changes from 3 to 30.

observation is that the average errors gets large when the radio range gets large and the network is better connected. This is because the measurement of hop-counts becomes "coarse" when the radio range becomes large.

Fig. 3 gives the average errors versus the ratio of beacon nodes for two different values of radio ranges ($r = u$ and $r = 1.3u$). In the case of a less connected network (Fig. 3 (a), $r = u$), PDM gives better estimation accuracy in all cases, and its accuracy significantly improves as the ratio of beacon nodes increases. (This is not the case with DV-hop.) In the case of a stronger connected network (Fig. 3 (b), r=1.3u), both PDM and DV-hop perform better than in the case of a less connected network. The performance of DV-hop is comparable to that of PDM in isotropic networks (Topology $\mathcal{A}$), but becomes much worse in anisotropic networks (Topology $\mathcal{B}$ and $\mathcal{C}$).

In summary, PDM and DV-hop use exactly the same information, i.e. the hop-counts and geographic distances between beacon nodes. However, PDM gives significantly better estimation accuracy in anisotropic networks.

V. Distributed Localization System based on PDM

A. Procedure for Information Collection and Linear Transformation Calculation

Localization in sensor networks is carried out at an initialization phase and further can be repeated during the lifetime of sensor networks. For example, if probing packets originated from a beacon node are dropped at a lossy wireless channel and cannot reach sensor nodes in a region, it is necessary for the beacon to perform the probing task of sending probing packets again. If a new beacon node is added after the initial deployment, some tasks for localization need to be repeated so as to exploit the location information of the newly deployed beacon node. To support various scenarios, we develop the following criteria for the protocol design of a distributed localization system: (1) a beacon node can initiate a probing procedure at any instant so as to notify other nodes in sensor networks of its existence, and (2) without a prior knowledge of the number of beacon nodes, beacon and sensor nodes can properly perform localization by the use of gradually discovered information.

The procedure for collecting geographic distance and proximity information between nodes is similar to that of *Adhoc Positioning System* (APS) [3]. Here, we pay more attention to how to minimize the communication and computational overheads. The procedure is as follows:

- **(P1)** Every node initializes an empty beacon list, whose entry will be filled with the location and the proximity for beacon nodes.
- **(P2)** After a random delay d_i $(0 < d_i \leq D_{init})$, each beacon node broadcasts to its neighboring nodes a probing packet containing its ID, location, and the "initial" proximity $\{i,\ \mathbf{x}_i,\ p_i = 0\}$.
- **(P3)** Whenever a node receives a probing packet, it calculates the new proximity. If the new proximity is larger than the proximity in the beacon list, the node discards the probing

packet. Otherwise, the node updates its beacon list and forwards the packet to its neighboring nodes.

Note that the proximity can be either the hop-count or the geographic distance measured using radio signal. If the proximity is measured as the hop-count, the proximity is increased by one for each hop. On the other hand, if the proximity is measured with the use of radio signals from ranging devices, it is increased by the measured geographic distance.

- **(P4)** If a beacon node b receives a probing packet containing the information for other beacon nodes, it performs (P3) as other nodes do, and updates the proximity vector $\mathbf{p}_b$. In addition, it informs the other beacon nodes of its updated $\mathbf{p}_b$ when there is no more probing packet from other beacon nodes during a time interval $D_{update} = D_{init} + D_{rtt}$, where D_{rtt} is the maximum round-trip-time in the sensor networks.

  In an exceptional case, if a beacon node receives no probing packet, (e.g., newly deployed beacon node), it computes its proximity vector $\mathbf{p}_b$ by averaging the proximity vectors of neighboring sensor nodes within its radio range.
- **(P5)** Whenever a beacon node b receives an update packet containing the updated $\mathbf{p}_b$ information, it updates both its proximity matrix $\mathbf{P}$ and geographic distance matrix $\mathbf{L}$. If there is no more update packet during a time interval $D_{svd} = 2D_{update} + D_{rtt}$, the beacon node b computes SVD of $\mathbf{P}$ and obtains $\mathbf{T}$ by Eq. (2).

  In particular, once the SVD of $\mathbf{P}$ is computed, it is not desirable to re-compute SVD of $\mathbf{P}$ because the computational complexity of SVD is $O(M^3)$. Instead, an incremental SVD technique proposed in [19] is used to update $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}$. With computational complexity $O(M^2)$, the incremental technique projects the new proximity vector onto the current SVD and obtains its approximation.
- **(P6)** A sensor node s obtains the proximity vector $\mathbf{p}_s$ from its beacon list, retrieves $\mathbf{T}$ from one of the beacon nodes, calculates the geographic distances to beacon nodes by Eq. (6), and estimates its location $\mathbf{x}_s$ by a lateration algorithm.

In comparison with APS, the additional complexity incurred in this procedure is (1) the communication overhead for collaboration of beacon nodes on constructing $\mathbf{P}$ ((P4)), and (2) the computational overhead for calculating the SVD of $\mathbf{P}$ so as to obtain $\mathbf{T}$ ((P5)). However, we claim that the added complexity is not significant because (1) in many cases each beacon

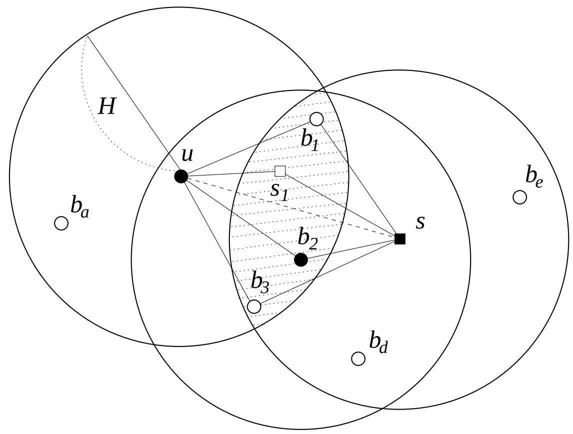


Fig. 4. Extrapolation of proximity in the case that the scope of packet flooding is limited (circle: beacon node, square: sensor node).

node needs only $(M-1)$ additional unicast packets to exchange its proximity vector with the other beacon nodes, and (2) the SVD of $\mathbf{P}$ can be incrementally computed, and its dimension is not large (i.e., M-by-M rather than $(M+N)$-by-$(M+N)$ as in MDS-based approaches).

B. Alternatives to Packet Flooding

The packet flooding by beacon nodes can overwhelm the sensor network. If the number of beacon nodes is sufficiently large, PDM can characterize the relationship of the geographic distance and the proximity between sensor nodes without packet flooding. If packet flooding is not used, (P2) and (P3) can be modified so that (1) all the beacon nodes unicast their probing packets to one another; and (2) a sensor node with an unknown location unicasts its probing packets to the beacon nodes and obtains its proximity vector. (We will show simulation results for this case in Section VI-B.)

Alternatively, we can simply limit the scope of packet flooding with specifying the TTL value to H-hops in each probing packet. One issue of this approach is that it may cause inconsistency between the beacon lists maintained by a beacon node b and a sensor node that contacting beacon node b. Figure 4 illustrates such an example. A sensor node s with an unknown location measures its proximities to a set of beacon nodes, i.e., B_s = $\{b_1$, b_2, b_3, b_d, $b_e\}$. In (P6), node s contacts the closest beacon node b_2 to obtain $\mathbf{T}_{b_2}$. As node b_2's beacon list is B_{b_2} = $\{b_1$, b_2, b_3, b_d, $u\}$ and $B_{b_2} - B_s$ = $\{u\}$, node s is required to estimate the proximity to beacon node u in order to use $\mathbf{T}_{b_2}$.

The above issue can be resolved as follows. Node s computes the proximity to node u, with the help of a set, $\bar{B}$, of nodes whose hop-counts to both nodes u and s are less than H, (e.g., $\bar{B}$ = $\{b_1$, b_2, b_3, $s_1\}$ in Fig. 4). As each node $n_i \in \bar{B}$ has the proximities from itself to both nodes u and s for $i = 1, \cdots, |\bar{B}|$, the proximity p_{su} can be approximately obtained as

$$p_{su} = \min_i \left(p_{sn_i} + p_{n_i u}\right).$$

If node s needs the geographic distances to the other beacon nodes (e.g., the beacon node b_e) for lateration, it repeats the above approach to the others.

C. Lateration Algorithm

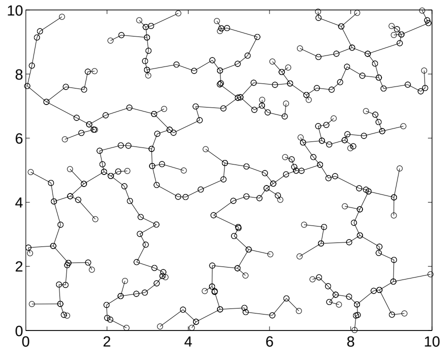
After (P6) is performed, each sensor node obtains the estimates of its geographic distances to beacon nodes. A lateration algorithm is required to determine the location of the sensor node. We consider the following lateration algorithms:

- (L1) Linearized model based method [3]: A linear system is derived by linearizing the the Euclidean distance equations with respect to *a priori* location estimate. The location is later corrected by a least square solution of the linear system. This correction process is iteratively performed by updating the linear system with the new location estimate.
- (L2) Descent gradient method [7]: The descent gradient method with a constant step size α is applied to minimize the objective function

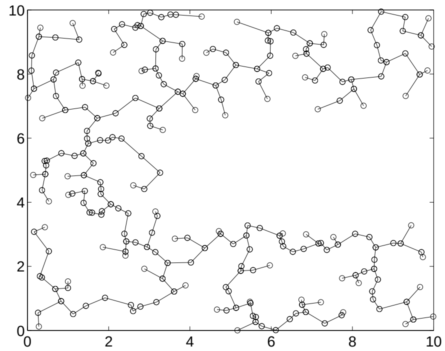
$$\mathcal{E} = \frac{1}{2}\sum_{i=1}^{M}(d_{si} - \tilde{l}_{si})^2.$$

- (L3) Non-iterative multilateration [4], [20]: For the quadratic version of the Euclidean distance equations, a linear system is derived by subtracting one of the equations from the other equations. The location estimate is given by the least square solution of the linear system.

Through simulation studies, we observe that (L1) gives relatively accurate estimates, but requires an initial guess of the location. The performance of the descent gradient method (L2) is susceptible to the step size α, the selection of which is not a trivial issue. A large step size causes divergence (especially for a large number of beacon nodes), while a small step size causes slow convergence. Non-iterative multilateration (L3) requires neither a judicious guess of



(a) Topology $\mathcal{A}$



(b) Topology $\mathcal{B}$

Fig. 5. Resulting network topologies after power control is applied to Topologies $\mathcal{A}$ and $\mathcal{B}$ in Fig. 1.

the initial location nor the time-consuming iteration process. However, its performance is more susceptible to the measurement noise than the others. As a result, we use the linearized model based lateration (L1) to determine the location of a sensor node, with the initial location set to the location of the beacon node that is closest (in terms of the estimated Euclidean distance) to the sensor node.

VI. Experimental Results

To evaluate the localization performance of PDM, we have conducted a simulation study. We compare the performances of DV-hop (DV-distance), MDS-map, and the proposed PDM-based method in the network configurations depicted in Figure 1. Note that unlike DV-hop and PDM, MDS-map requires the global information between all the sensor nodes. We consider two sets of scenarios according to how the proximity information is obtained:

(A) the proximity information is gathered by packet flooding (Section V-A); and

(B) power control is first applied so that each sensor node does not transmit with the maximal transmission power, but instead an adequate transmission power range to maintain network connectivity. As a result, different sensor nodes may have different radio ranges. Moreover, the proximity information is disseminated via unicasts (to the other beacon nodes; Section V-B), rather than packet flooding.

Figure 5 depicts the resulting network topology after power control has been applied to Topologies $\mathcal{A}$ and $\mathcal{B}$. In this set of scenarios, as each sensor node has a different radio range, the product of the hop-count and the radio range is no longer a proper estimate of the geographic distance. This implies that the relationship between the geographic distance and the proximity is more complicated in this scenario than in scenario A.

In each experiment, two types of proximity are considered: the hop-count and the estimated geographic distance between sensor nodes (obtained from the received radio signal strength). Also, the following error index is used to quantify the estimation error:

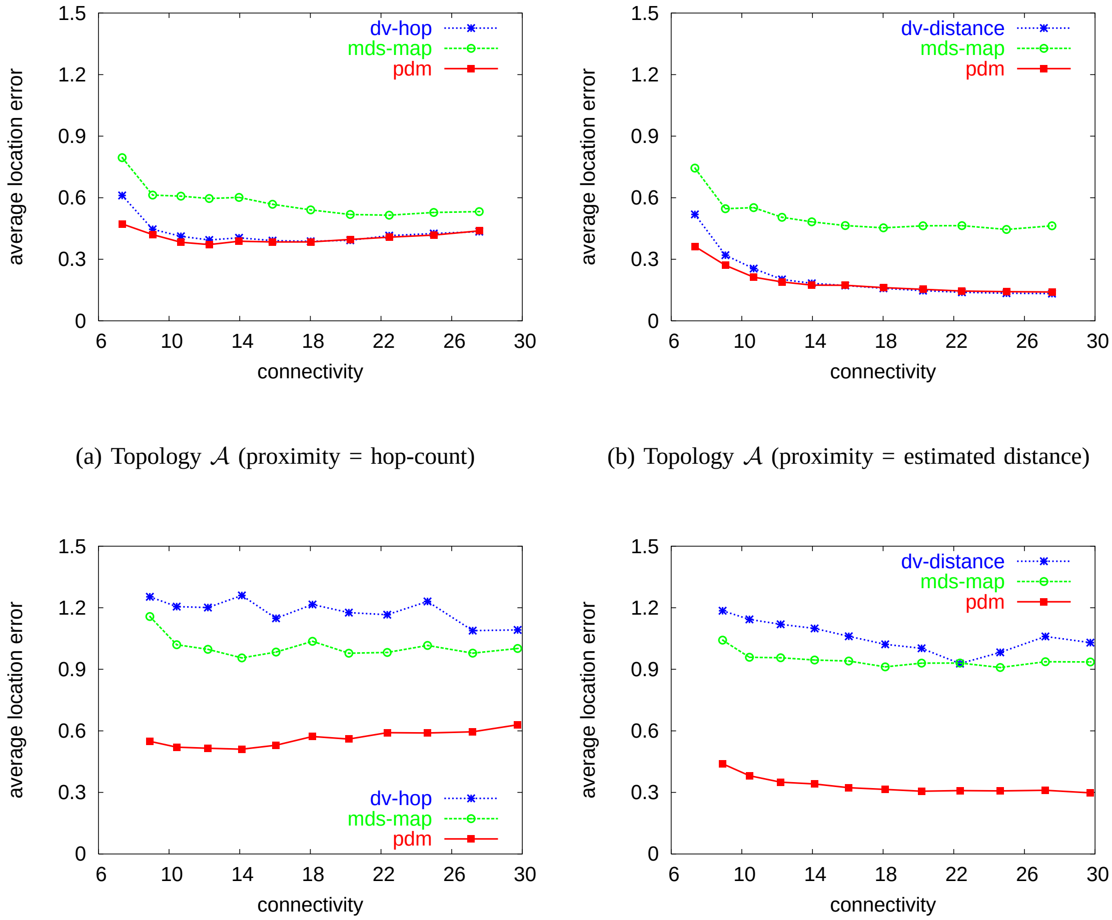
$$\mathcal{E}_p = \frac{\sum_{i=1}^{M+N} \sum_{j=1}^{M+N} f_d(\mathbf{x}_i, \mathbf{x}_j)}{(M+N)^2 \ u},$$

where $M + N$ is the total number of sensor nodes.

A. Results for Scenario A

Effect of radio ranges on localization accuracy*:* First we investigate the effect of radio ranges (or equivalently how well the network is connected) on the localization accuracy in isotropic (Topology $\mathcal{A}$) and anisotropic (Topology $\mathcal{B}$) networks. If sensor nodes are uniformly distributed in a square area and well connected, the network is considered to be isotropic. Figure 6 (a) and (b) depict the average location errors in isotropic networks (Topology $\mathcal{A}$). PDM gives the smallest estimation error under all cases. DV-hop performs as well as PDM when sensor nodes are well connected ($r \geq 1.4u$). If the hop-count is used as the proximity measure (Fig. 6 (a)), the performance of all three methods (in terms of estimation accuracy) slightly deteriorates as the radio range increases. This is because the proximity is expressed as coarser integer values. On the other hand, if the estimated geographic distance is used as the proximity measure (Fig. 6 (b)), the performance improves as the radio range increases. Figure 6 (c) and (d) depict the average location errors in anisotropic networks (Topology $\mathcal{B}$). The estimation error of PDM is, respectively, half and one-third of that of MDS-map and DV-hop.

Effect of the number of beacon nodes on localization accuracy*:* Second we investigate the effect of the number of beacon nodes on the localization accuracy in isotropic (Topology $\mathcal{A}$) and anisotropic (Topologies $\mathcal{B}$ and $\mathcal{C}$) networks. We vary the number of beacon node M from 4 to 30. In the case of M = 30, the ratio of beacon nodes to the total number of sensors is 0.12.



(a) Topology $\mathcal{A}$ (proximity = hop-count)

(b) Topology $\mathcal{A}$ (proximity = estimated distance)

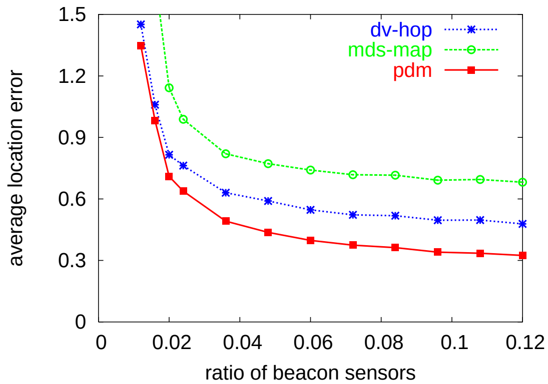
(c) Topology $\mathcal{B}$ (proximity = hop-count)

(d) Topology $\mathcal{B}$ (proximity = estimated distance)

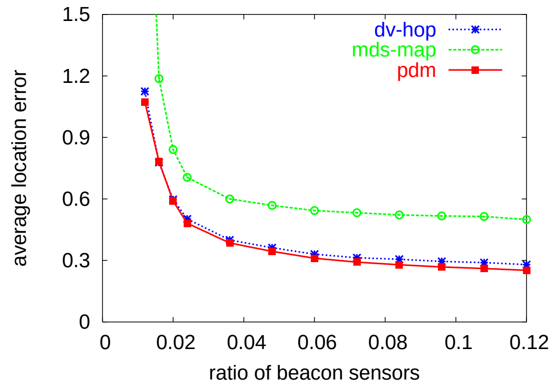
Fig. 6. Localization errors versus radio ranges ($u \leq r \leq 2u$), when the number of beacon node is 10. The x-axis is the number of neighbors (as r changes from u to $2u$).

Figure 7 gives the localization errors when the hop-count is used as the proximity measure. In the case of isotropic networks (Topology $\mathcal{A}$), if the radio range is large enough to provide strong connectivity, (e.g., $r = 1.3u$ in Fig. 7 (b)), DV-hop and PDM give almost the same performance. In the case of anisotropic networks (Topologies $\mathcal{B}$ and $\mathcal{C}$), DV-hop and MDS-map perform comparatively worse than PDM (Fig. 7 (c)–(f)).

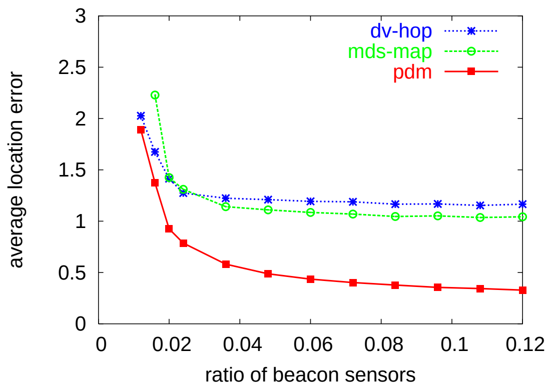
Figure 8 gives the localization errors when the estimated Euclidean distance is used as the the proximity measure. As compared to Fig. 7, the performance of all three methods significantly improves, and PDM gives the smallest estimation errors.



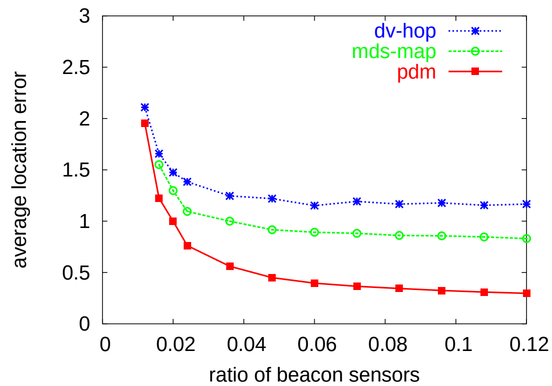
(a) Topology $\mathcal{A}$ ($r = u$)



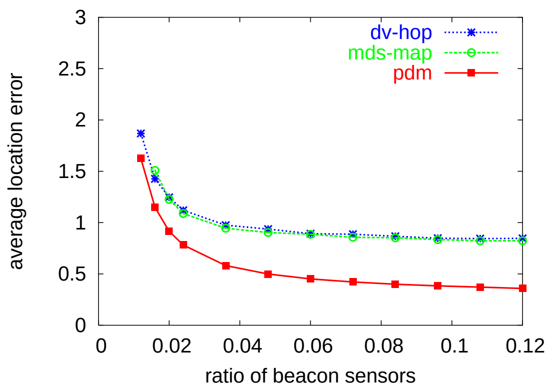
(b) Topology $\mathcal{A}$ ($r = 1.3u$)



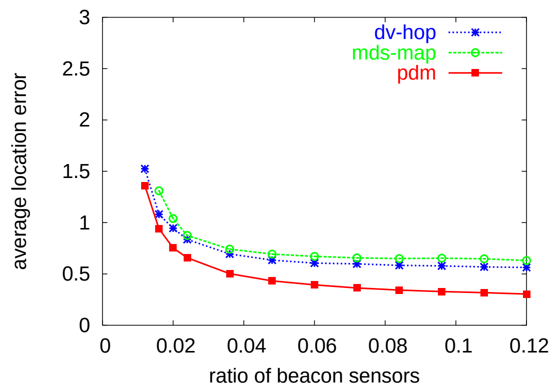
(c) Topology $\mathcal{B}$ ($r = u$)



(d) Topology $\mathcal{B}$ ($r = 1.3u$)

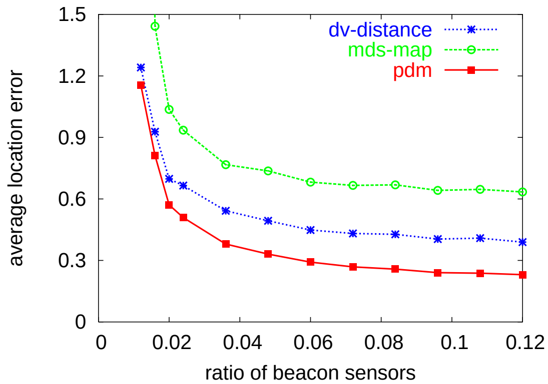


(e) Topology $\mathcal{C}$ ($r_1 = u$, $r_2 = 1.3u$)

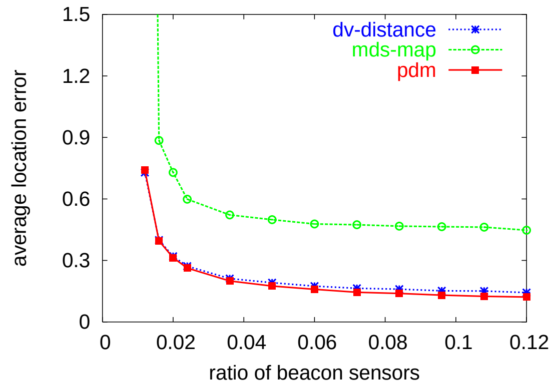


(f) Topology $\mathcal{C}$ ($r_1 = 1.3u$, $r_2 = 1.69u$)

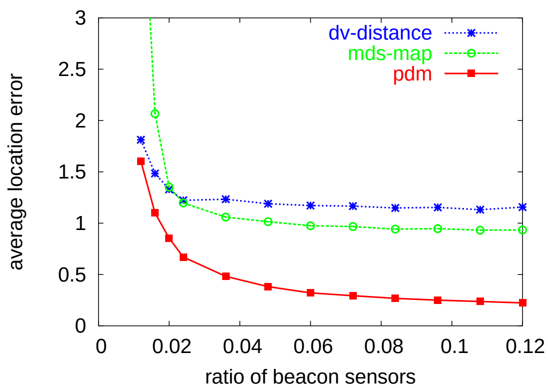
Fig. 7. Localization errors versus the ratio of beacon nodes. The hop-count is used as the proximity metric.



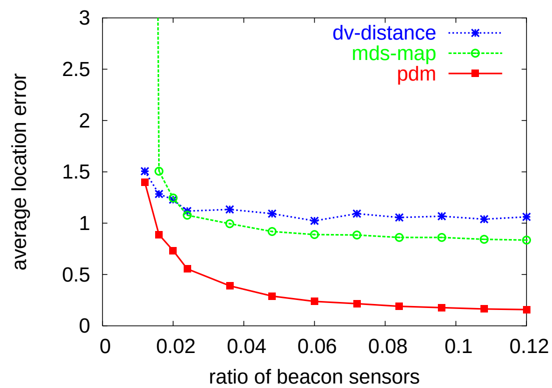
(a) Topology $\mathcal{A}$ ($r = u$)



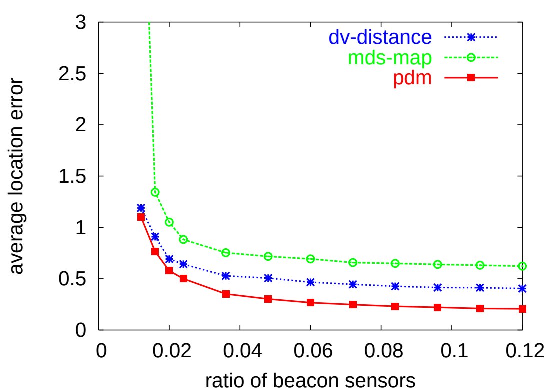
(b) Topology $\mathcal{A}$ ($r = 1.3u$)



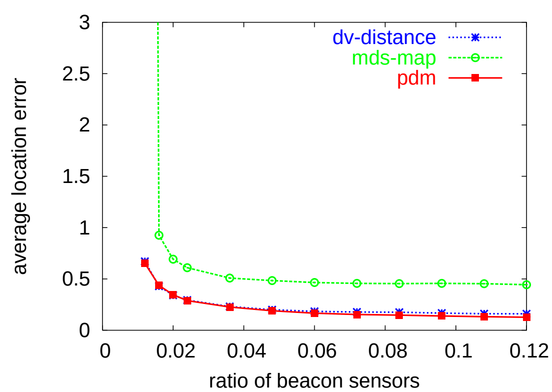
(c) Topology $\mathcal{B}$ ($r = u$)



(d) Topology $\mathcal{B}$ ($r = 1.3u$)

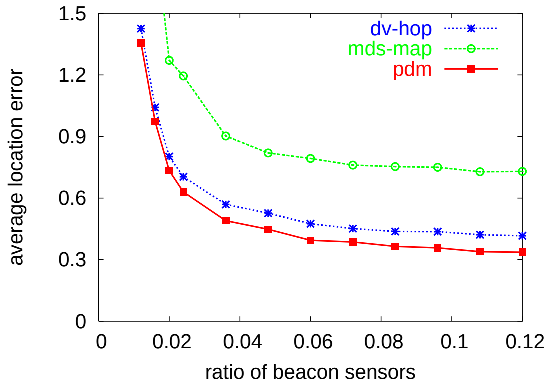


(e) Topology $\mathcal{C}$ ($r_1 = u$, $r_2 = 1.3u$)

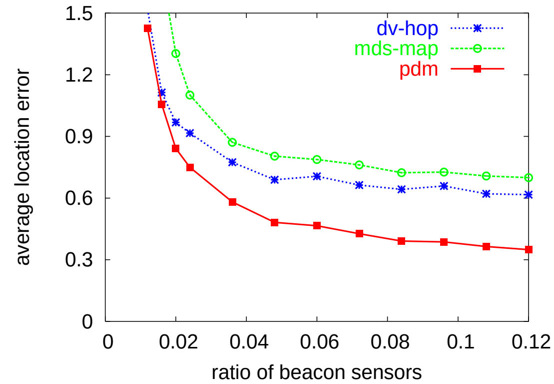


(f) Topology $\mathcal{C}$ ($r_1 = 1.3u$, $r_2 = 1.69u$)

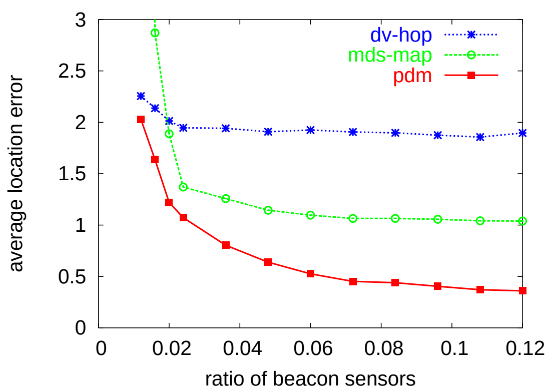
Fig. 8. Localization errors versus the ratio of beacon nodes. The estimated geographic distance is used as the proximity metric.



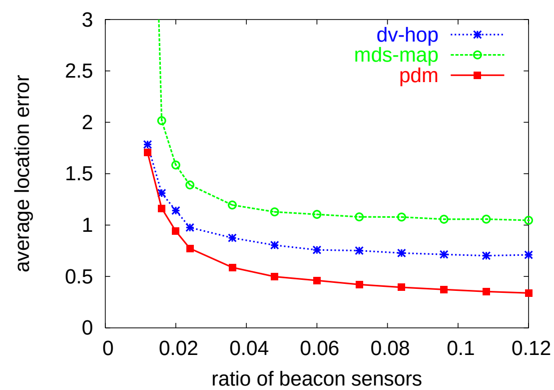
(a) Topology $\mathcal{A}$ (denser at right half)



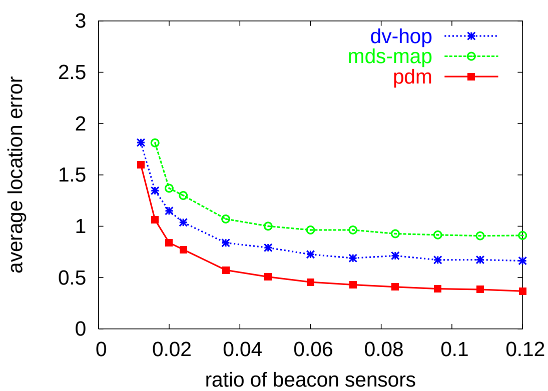
(b) Topology $\mathcal{A}$ (denser at left half)



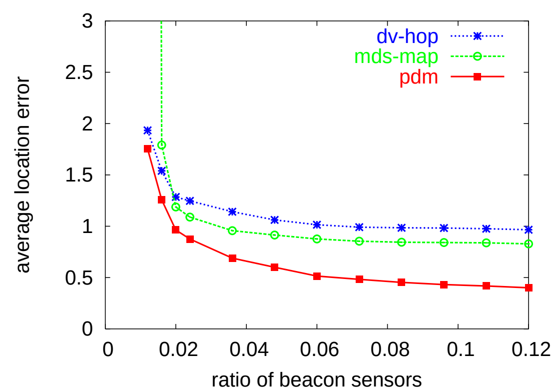
(c) Topology $\mathcal{B}$ (denser at right half)



(d) Topology $\mathcal{B}$ (denser at left half)



(e) Topology $\mathcal{C}$ (denser at right half)



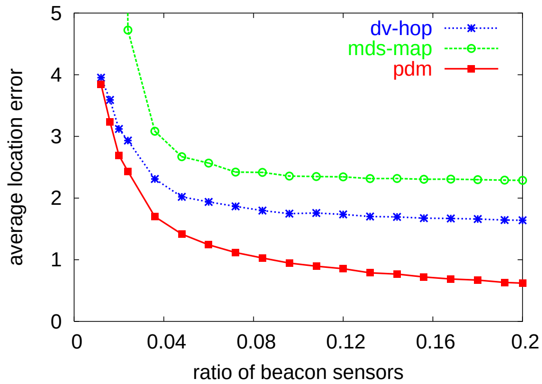
(f) Topology $\mathcal{C}$ (denser at left half)

Fig. 9. Localization errors versus the ratio of beacon nodes, when beacon nodes are not uniformly distributed. The density of beacon nodes is either linearly increasing (left column) or decreasing (right column) with respect to the x-axis. The hop-count is used as the proximity metric.

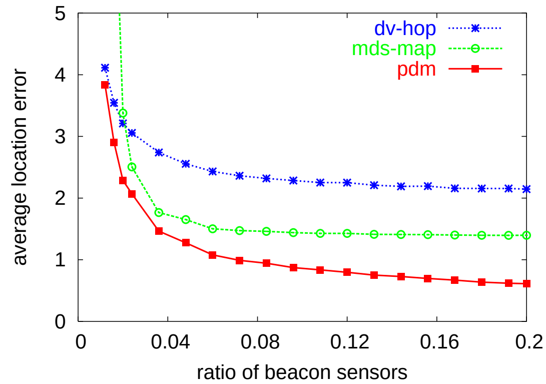
Effect of the non-uniform distribution of beacon nodes on localization accuracy*:* Third we investigate the effect of the non-uniform distribution of beacon nodes in isotropic (Topology $\mathcal{A}$ (r = u)) and anisotropic (Topology $\mathcal{B}$ (r = u) and $\mathcal{C}$ (r_1 = u, r_2 = 1.3u)) networks. In order to obtain the non-uniform distribution of beacon nodes, we choose M beacon nodes from sensor nodes according to a probability p_b, which either linearly increases or decreases with respect to the x-axis, i.e., $p_b = 2r_b x/x_{max}$ or $p_b = -2r_b(x/x_{max} - 1)$, where r_b, x, and x_{max} are the ratio of beacon nodes to sensor nodes, the x coordinate value for a selected node, and the width of sensor network area, respectively.

Figure 9 gives the localization errors when the beacon node are non-uniformly distributed. As the hop-count is used as the proximity metric, the location errors can be compared with those for the uniform distribution in Fig. 7 (a), (c), and (e). In Topology $\mathcal{A}$, the location errors for DV-hop is the smallest when the density of beacon nodes increases with respect to the x-axis in Fig. 9 (a). This is because the deployment of the sensor nodes is slightly denser in right half plane as shown in Fig. 1 (a). MDS-map and PDM give almost the same performance regardless of the distribution of beacon nodes. In Topologies $\mathcal{B}$ and $\mathcal{C}$, the accuracy of DV-hop is apparently affected by the distribution of beacon nodes. The localization errors of DV-hop for Topologies $\mathcal{B}$ and $\mathcal{C}$ are the smaller in Fig. 9 (d) and (e) than in Fig. 9 (c) and (f), respectively. Moreover, the errors are even smaller than those in the case of the uniform distribution in Fig. 7 (c) and (e), respectively. This implies that the locations of beacon nodes for DV-hop should be carefully determined so as to gather topological information accurately. In contrast, we observe that the performances of MDS-map and PDM are less sensitive to the distribution of beacon nodes in both isotropic and anisotropic networks.

In summary, DV-hop (DV-distance) gives accurate estimates of geographic locations only in isotropic networks with high connectivity. MDS-map gives better performance than DV-hop (DV-distance) in Topology $\mathcal{B}$, perhaps due to the fact that the two eigenvectors obtained by MDS capture principal components of anisotropic properties. PDM achieves the best performance consistently. As the radio range and the number of beacon nodes are larger than certain thresholds, the estimation errors for PDM fall below 0.3u under all cases.



(a) Topology $\mathcal{A}$



(b) Topology $\mathcal{B}$

Fig. 10. Localization errors versus the ratio of beacon nodes in power-controlled sensor networks. The hop-count is used as the proximity metric.

B. Results for Scenario B

Effect of the number of beacon nodes on localization accuracy*:* Figure 10 shows the localization errors when the hop count is used as the proximity measure. We vary the number of beacon nodes M from 4 to 50. In the case of M = 50, the ratio of the number of beacon nodes to the total number of nodes is 0.2. As shown in Fig. 10 (a), the estimation errors for DV-hop and MDS-map are quite large and decrease slowly as the number of beacon nodes increases. The estimation errors for PDM are much smaller and decrease faster as the number of beacon nodes increases. This implies that PDM can capture the topological features in the case of low connectivity (as power control has been applied). We observe that the error falls below 1 and 0.5 for $M > 14$ (ratio = 0.056) and 41 (ratio = 0.164), respectively. In Fig. 10 (b), DV-hop and MDS-map do not show performance improvement when $M > 25$ (ratio = 0.1), while PDM gives almost the same improvement as that in Fig. 10 (a).

In summary, it is more difficult to extract the geographic information from proximity measurements, after sensor nodes exercise power control and the network connectivity becomes lower. Even under the case, PDM makes accurate location estimation as long as the ratio of beacon nodes exceeds a certain threshold.

VII. Conclusion

In this paper, we have designed and evaluated a new PDM-based localization method in anisotropic sensor networks. We represent the measured proximities and the geographic distances in Lipschitz embedding spaces, and devise a transformation method that projects the coordinates in the embedding space built on proximities into the geographic distance space. The transformation matrix accurately characterizes anisotropic network topologies because it retains the components of proximities to the beacon nodes in *all* directions. We show that the transformation can be obtained by using the truncated SVD pseudo-inverse technique even in the presence of measurement noises. Finally, we show via simulation that the proposed PDM-based method outperforms DV-hop, DV-distance [3], and MDS-map [10], and makes robust and accurate estimates of sensor locations in both isotropic and anisotropic sensor networks.

Recall that in Section V-B, we state that if the number of beacon nodes is sufficiently large, each (beacon or non-beacon) node can measure its proximities to a subset of beacon nodes by unicast probing. The criteria for correctness are (1) all the beacon nodes in a subset unicast their probing packets to one another; and (2) an unknown node obtains the proximity-distance map (PDM) from one of the beacon nodes in the same subset. As part of our future work, we will investigate how to decompose the set of beacon nodes into subsets, so that proximity measurements among beacon nodes in each subset are sufficient to capture the anisotropic characteristics of the entire sensor network.

REFERENCES

[1] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-less low-cost outdoor localization for very small devices," *IEEE Personal Communications*, pp. 28–34, 2000.

[2] L. Doherty, K. Pister, and L. Ghaoui, "Convex position estimation in wireless sensor networks," in *Proceedings of IEEE Infocom*, 2001.

[3] D. Niculescu and B. Nath, "Ad hoc positioning system (APS)," in *Proceedings of IEEE Globecom*, 2001.

[4] A. Savvides, C. C. Han, and M. B. Srivastava, "Dynamic fine-grained localization in ad-hoc wireless sensor networks," in *Proceedings of ACM Mobicom*, 2001.

[5] C. Savarese, J. Rabaey, and K. Langendoen, "Robust positioning algorithms for distributed ad-hoc wireless sensor networks," in *Proceedings of USENIX Technical Annual Conference*, 2002.

[6] A. Savvides, H. Park, and M. B. Srivastava, "The bits and flops of the N-hop multilateration primitive for node localization problems," in *Proceedings of WSNA*, 2002.

[7] R. Nagpal, H. Shrobe, and J. Bachrach, "Organizing a global coordinate system from local information on an ad hoc sensor network," in *Proceedings of IPSN*, 2003.

[8] T. He, C. Huang, B. M. Blum, J A. Stankovic, and T. Abdelzaher, "Range-free localization schemes for large scale sensor networks," in *Proceedings of ACM Mobicom*, 2003.

[9] V. Raykar, I. Kozintsev, and R. Lienhart, "Position calibration of audio sensors and actuators in a distributed computing platform," in *Proceedings of ACM Multimedia*, 2003.

[10] Y. Shang, W. Ruml, and Y. Zhang, "Localization from mere connectivity," in *Proceedings of ACM Mobihoc*, 2003.

[11] Y. Shang and W. Ruml, "Improved MDS-based localization," in *Proceedings of IEEE Infocom*, 2004.

[12] X. Ji and H. Zha, "Sensor positioning in wireless ad-hoc sensor networks using multidimensional scaling," in *Proceedings of IEEE Infocom*, 2004.

[13] G. R. Hjaltason and H. Samet, "Properties of embedding methods for similarity searching in metric spaces," *IEEE Trans. on Pattern Analysis and machine intelligence*, vol. 25, no. 5, pp. 530–549, 2003.

[14] L. Tang and M. Crovella, "Virtual landmarks for the Internet," in *Proceedings of ACM Internet Measurement Conference*, 2003.

[15] H. Lim, J. C. Hou, and C.-H. Choi, "Constructing Internet coordinate system based on delay measurement," in *Proceedings of ACM Internet Measurement Conference*, 2003.

[16] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, 1996.

[17] Y. S. Shim and Z. H. Cho, "SVD pseudo-inversion image reconstruction," *IEEE Trans. on Acoust, Speech, Signal Processing*, vol. ASSP-29, 1981.

[18] N. G. Gencer and S. J. Williamson, "Differential characterization of neural sources with the bimodal truncated SVD pseudo-inverse for EEG and MEG measurements," *IEEE Trans. on Biomedical Engineering*, vol. 45, no. 7, 1998.

[19] B. Sarwar, G. Karypis, J. Konstan, and J. Riedl, "Incremental SVD-based algorithms for highly scalable recommender systems," in *Proceedings of International Conference on Computer and Information Technology*, 2002.

[20] K. Langendoen and N. Reijers, "Distributed localization in wireless sensor networks: a quantitative comparison," *Elsevier Computer Networks*, vol. 43, pp. 499–518, 2003.